MATH 4220 HOMEWORK 0 SOLUTIONS

Exercise 3: By (i), we must have either $i \in \mathcal{P}$ or $-i \in \mathcal{P}$; suppose the former. Then $-i = i^3 \in \mathcal{P}$ by (iii), contradicting (i). Similarly, if $-i \in \mathcal{P}$, then $(-i)^3 = i \in \mathcal{P}$ by (iii), again contradicting (i).

Exercise 4: For $z = x + iy$ and $w = u + iv$, we have

$$\overline{zw} = \overline{(xu - vy) + i(yu + xv)} = (xu - vy) - i(yu + xv),$$

$$\overline{z}\overline{w} = (x - iy)(u - iv) = (xu - yv) - i(yu + xv),$$

so $\overline{zw} = \overline{z}\overline{w}$. By induction, we have $\overline{z}^k = \overline{z^k}$ whenever $k \geq 0$. Applying this to z^{-1} in place of z and using $\overline{z}^{-1} = \overline{z^{-1}}$ gives the claim when $k < 0$ (assuming $z \neq 0$).

Exercise 5: The equation $A + B = z$ is as follows:

$$(a\cos(\theta_1) + b\cos(\theta_2)) + i\,(a\sin(\theta_1) + b\sin(\theta_2)) = l + id,$$

so that $d = a\sin(\theta_1) + b\sin(\theta_2)$ and $l = a\cos(\theta_1) + b\cos(\theta_2)$. We now solve for θ_2 in terms of θ_1 and d:

$$\theta_2 = \cos^{-1}\left(\frac{l - a\cos(\theta_1)}{b}\right).$$

Combining equations gives the desired formula.