Math 4220: Homework 3

This is the third homework assignment for Math 4220. I have broken the homework assignment into two parts. Part I has exercises that you should do (and I expect you to do) but you need not turn in. The exercises in the second part of the homework are exercises you should write up and submit via Gradescope. Your solutions should be consistent with the directions in the syllabus. If you get stuck on any part of the homework, please come and see me or Max. More importantly, have fun!

Part I (Do not write up)

Exercise 1. Read Sections 2.4, 2.5, 2.6, and 2.7 of the course textbook.

Exercise 2. Do Exercises 1, 2, 3, 5, 7, 8, 13, and 15 in Section 2.4 of the textbook.

Exercise 3. Do Exercises 1, 2, 3, 5, 6, 7, 9, 15, and 21 in Section 2.5 of the textbook.

Exercise 4. Do Exercises 1 and 3 in Section 2.6 of the textbook.

Exercise 5. Do Exercises 1, 4, 5, and 9 in Section 2.7 of the textbook.

Part II (Write up and Submit via Gradescope)

Exercise 1. Please do Exercise 4 in Section 2.4 of the course textbook.

Exercise 2. Please do Exercise 6 in Section 2.4 of the course textbook.

Exercise 3. Please do Exercise 14 in Section 2.4 of the course textbook.

Exercise 4.

1. Please do Exercise 14 in Section 2.5 of the course textbook.
2. Use the result of Exercise 14 to produce a solution to the following boundary problem for Laplace's equation: Find a function $u(z) = u(x, y)$ for which

$$\begin{cases} \Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 & \text{for } (x,y) \in A \\ u(x,y) = 0 & \text{for } (x,y) \in S_1 \\ u(x,y) = 1 & \text{for } (x,y) \in S_2. \end{cases}$$

where $A = \left\{(x,y) : 1 < \sqrt{x^2+y^2} < 2\right\}$, $S_1 = \left\{(x,y) : \sqrt{x^2+y^2} = 1\right\}$ and $S_2 = \left\{(x,y) : \sqrt{x^2+y^2} = 2\right\}$.
Hint: I gave a solution to this boundary value problem in class.

Exercise 5.

1. Solve the following boundary value problem: Find a function $u(z) = u(x, y)$ for which

$$\begin{cases} \Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 & \text{for } (x,y) \in G \\ u(x,y) = 2 & \text{for } (x,y) \in L_0 \\ u(x,y) = 10 & \text{for } (x,y) \in L_1. \end{cases}$$

where $G = \{z = x + iy : 0 < \mathrm{Re}(z) < 1\}$, $L_0 = \{z = x + iy : \mathrm{Re}(z) = 0\}$, and $L_1 = \{z = x + iy : \mathrm{Re}(z) = 1\}$.

2. Find the function f which is analytic on G for which $\mathrm{Re}(f) = u$.
Hint: You can find u by any method you want. However, it is helpful to look for f first.

Exercise 6. Please do Exercise 2 in Section 2.6 of the course textbook.