Math 4220: Homework 5

This is the fifth homework assignment for Math 4220. I have broken the homework assignment into two parts. Part I has exercises that you should do (and I expect you to do) but you need not turn in. The exercises in the second part of the homework are exercises you should write up and submit via Gradescope. Your solutions should be consistent with the directions in the syllabus. If you get stuck on any part of the homework, please come and see me or Max. More importantly, have fun!

Part I (Do not write up)

Exercise 1. Read Sections 3.4, 3.5, and 4.1 of the course textbook.

Exercise 2. Do Exercises 1, 3, 5, and 7 in Section 3.4 of the textbook.

Exercise 3. Do Exercises 1, 3, 5, 7, 11, and 15 in Section 3.5 of the textbook.

Exercise 4. Do Exercises 1,3, 5, 7, 9, 11, and 13 in Section 4.1 of the textbook.

Part II (Write up and Submit via Gradescope)

Exercise 1. Please do Exercise 2 in Section 3.4 of the course textbook.

Exercise 2. Please do Exercise 9 in Section 3.5 of the course textbook.

Exercise 3. Please do Exercise 12 in Section 3.5 of the course textbook.

Exercise 4. Please do Exercise 16 in Section 3.5 of the course textbook.

Exercise 5. Please do Exercise 10 in Section 4.1 of the course textbook.

Exercise 6.

1. Please do Exercise 14 of Section 4.1. In the course of your argument/verification, please make sure to explain where exactly that hypotheses (i) and (ii) are used.

2. Consider now the admissible parameterization $z_1 : [0, 1] \to \mathbb{C}$ defined by $z_1(t) = e^{t2\pi i}$ of the unit circle where $a = 0$ and $b = 1$ and also consider $z_2(t) = z_1(\phi(t))$ where $\phi(t) = -t$ and so $c = 0$ and $d = -1$. In this case, compute
$$\int_a^b |z_1'(t)|\; dt$$
and
$$\int_c^d |z_2'(s)|\; ds$$
Are these quantities equal? If not, in view of the statement given in Exercise 14, explain why that statement didn't apply here, i.e., explain why we didn't meet the hypotheses.

3. In light of what you just found, discuss why the length of a smooth curve γ (defined in (1)), while invariant under changes of parameterization (as in Exercise 14), is not invariant under changes to "orientation".