As promised, this is an itemized list of topics that we've covered in MA4220 since the beginning of the semester. This list of topics (and the proportion of time we've spend on them since the beginning of the semester) will align with the problems you will see on Thursday's Prelim. In studying for Prelim, please note that I consider the homework exercises (both the "do not turn in" and the "turn in" exercises) and the examples I've covered in lecture to be the best source of practice problems. If you know how to approach each problem, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution, you should perform well on the exam.

1. You should know all basic properties and operations for complex numbers (including, for $z \in \mathbb{C}$, $\overline{z}$, $\mathrm{Re}(z)$, $\mathrm{Im}(z)$, $1/z$...)

2. You should know the definition of the complex modulus and its basic properties.

3. You should know the exponential/polar form of a non-zero complex number. Moreover, you should be able to convert between the standard (rectangular) and exponential forms.

4. For a given non-zero complex number $z$, you should know how to compute $\arg(z)$, Arg(z), and you should know the difference between these.

5. You should understand the geometry of the algebraic operations on $\mathbb{C}$. For example, given $z_1$ and $z_2$ in $\mathbb{C}$, the sum $z_1 + z_2$ is (geometrically) the vector sum. For non-zero elements $z_1, z_2 \in \mathbb{C}$, the product $z_1 z_2$ is the complex number whose modulus is $|z_1||z_2|$ and whose argument is $\arg(z_1) + \arg(z_2)$.

6. You should know how to find various integer roots of non-zero complex numbers. Also, you should know de Moivre's formula and how it's gotten.

7. You should know the basic point-set topology of $\mathbb{C}$ described in Section 1.6. In particular, know the definitions of: Neighborhoods/Balls, open sets, interior points, boundary points, boundary, closed sets, (path) connected sets, domain, region, and bounded/unbounded sets.

8. You should know the basic structure of functions of a complex variable. In particular, know that they can be expressed via their real and imaginary parts, $u$, $v$. You should know about the domain and range of a function and, if a function is given by a rule, what its associated natural domain is.

9. You should know (lots and lots!) about the exponential function. In particular, you should know its definition and basic properties. You should be able to compute $f(z) = e^z$ for various complex numbers $z$.

10. You should know the following definition:

    **Definition 1.** *Let $\{z_n\}$ be a sequence of complex numbers and let $w$ be a complex number. We say that the sequence $\{z_n\}$ converges to $w$ and write $\lim_{n\to\infty} z_n = w$ if, for all $\epsilon > 0$ there is a natural number $N$ for which*

    $$|z_n - w| < \epsilon \qquad \textit{whenever} \qquad n \geq N.$$

11. You should know the definition of the (complex) limit of a complex function. Recall:

    **Definition 2.** *Let $z_0 \in \mathbb{C}$ and let $f$ be a complex function whose domain $D$ containing $z_0$. We say that the limit of $f(z)$ as $z$ approaches $z_0$ exists and is equal to $w$ (written $\lim_{z\to z_0} f(z) = w$)) if, for every $\epsilon > 0$ there is a $\delta > 0$ for which*

    $$|f(z) - f(z_0)| < \epsilon \qquad \textit{whenever} \qquad 0 < |z - z_0| < \delta.$$

12. You should know how to use the above definition for various functions $f$ (as we did in class). You should also know how to show when a limit doesn't exist. For this, you may use the following proposition (proved in class) and its subsequent corollary.

**Proposition 3** (Proposition A.). *Let $z_0 \in \mathbb{C}$ and suppose that*

$$\lim_{z \to z_0} f(z) = w.$$

*The, for any admissible path/parameterization $\gamma : [0,1] \to \mathbb{C}$ for $f$ at $z_0$, i.e., $\gamma(t)$ is a continuous function with $\gamma(t) \neq z_0$ for all $t > 0$, $\gamma(t)$ lives in the domain of $f$ for all $t \in [0,1]$ and for which*

$$\lim_{t \searrow 0} \gamma(t) = z_0,$$

*we have*

$$\lim_{t \searrow 0} f(\gamma(t)) = w.$$

**Corollary 4.** *Let $z_0 \in \mathbb{C}$ and $f$ be a function which contains a deleted neighborhood of $z_0$. If there are two admissible paths $\gamma_1$ and $\gamma_2$ for $f$ at $z_0$ such that*

$$\lim_{t \searrow 0} f(\gamma_1(t)) \neq \lim_{t \searrow 0} f(\gamma_2(t))$$

*(where we take this to hold if one or both of the above limits don't exist), then the limit of $f$ at $z_0$ does not exist.*

13. You should know the basic theorems about limits, i.e., everything in Section 2.2.

14. You should know the definitions of various limits involving $\infty$; these are done at the end of Section 2.2 (but you also developed them in Homework 2).

15. You should know what it means for a complex function to be continuous at a point and on a set.

16. You should know the definition of the complex derivative of a function $f$ at $z_0$. Please know how to determine if a (relatively simple) function is differentiable by using the (limit) definition. You should know what it means for a function to be analytic on an open set. You should also know what it means for a function to be entire.

17. You should know that differentiable functions are continuous and have an idea why this is the case.

18. You should know the various differentiation formulas (product, quotient, sum, etc).

19. You should know the Cauchy-Riemann equations. You should especially know the Theorems in Section 2.4 concerning necessary and sufficient conditions for differentiability in terms of the Cauchy-Riemann equations. Note: We spent a lot of time on this, so it's worthy of really thinking through these theorems (including really understanding the hypotheses) and the associated proofs.

20. You should know Theorem 1 in Section 1.6 and Theorem 6 in Section 2.4, especially why the statements are false provided that $D$ is not a domain. Also, in looking through your notes, my proof of Theorem 1 involved the mean value theorem. Please understand how I used it.

21. You should know what harmonic functions are and you should know that the real and imaginary parts of analytic functions are harmonic. You should know about harmonic conjugates and the geometry of the level sets of harmonic conjgate pairs $u$ and $v$ (Section 2.5).

22. You should know about Boundary value problems for Laplace's equation that we discussed in lecture and you should master the related problems I solved in lecture and the material in Section 2.6.

23. You should know the material in Section 2.7.

24. You should know all about polynomials and rational functions. You should know the fundamental theorem of algebra. You should know everything in Section 3.1, though I will not ask you anything about Theorem 2 on page 105 on the exam.

25. Again, you should know ALL about the exponential function.

26. You should know about the logarithm (as a multi-valued function).

27. You should know about the various branches of logarithm, especially the principal branch of $\mathrm{Log}(z)$. You should know its properties, especially where it is continuous and analytic (and why).

28. You should know the definitions of the complex trig functions, especially $\sin(z)$, $\cos(z)$. You should know some things about these too. For example, can you prove that
$$\sin(z) = 0$$
if and only if $z = n\pi = n\pi + 0i$ for $n \in \mathbb{Z}$?

29. You should know how to compute derivatives of all the functions discussed above.

30. You should know how the function $z^\beta$ is defined when $\beta \in \mathbb{C}$. Note: It depends on the branch of log chosen. For example, you should know how to compute
$$z^\beta = e^{\beta \,\mathrm{Log}(z)}$$
for various values of $z$ and $\beta$.

31. You should know the material in Section 3.4.

32. You should know **all** of the material in Sections 4.1, 4.2 and 4.3.

33. You should know how to compute the length of a contour. You should know how to prove that the length is independent of parameterization.

34. Given a contour $\Gamma$, with parameterization $z = z(t)$, and a function $f$, you should know the definition of the contour integral
$$\int_\Gamma f(z)\,dz.$$
You should be able to compute contour integrals. For example, you should be able to work out (on your own) all of the examples in Section 4.2 and all of the examples I did in lecture.

35. You should know how to compute
$$\int_{C_r(z_0)} (z - z_0)^n\,dz$$
where $n \in \mathbb{Z}$ and $C_r(z_0)$ is the circle of radius $r > 0$ with center $z_0$.

36. You should know and know how to use the “TFAE” theorem on Page 176. You should thoroughly understand its proof.