

As promised, this is an itemized list of topics that we've covered in MA4220 since Prelim 1. This list of topics (and the proportion of time we've spend on them since the first prelim) will align with the problems you will see on Thursday's Prelim. Though Prelim 2 is not explicitly cumulative, per say, it will require you to know the material we've covered since the beginning of the course insofar as the recent material relies on it. Below this list of topics are some "do not turn in problems" for you to work through on the recent material. In studying for Prelim, please note that I consider the homework exercises (both the "do not turn in" and the "turn in" exercises) and the examples I've covered in lecture to be the best source of practice problems. If you know how to approach each problem, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution, you should perform well on the exam.

1. You know know Cauchy's integral formula and you should be very familiar with its proof.
2. As the derivative versions of Cauchy's integral formula rely on it, you should know Theorem 15 on Page 207 and be very comfortable with its proof.
3. You should know Cauchy's integral formula for derivatives (how it relies on Theorem 15) and also you should know the results in that section related to it, e.g., if a function is analytic, it is infinitely differentiable.
4. You should know Theorem 20 on Page 215, how it is proved via the derivative version of Cauchy's Integral Formula, and how Liouville's theorem (Theorem 21) follows from it.
5. You should know the Fundamental Theorem of Algebra and how to prove it.
6. You should know the maximum modulus principle and be very familiar with the proof I gave in class.
7. We spent a lot of time in Section 4.7 (Applications to Harmonic Functions). You should know all of the results we covered therein and be comfortable with the proofs and the applications (e.g., how to use the Poisson integral formula to solve the Diriclet problem on the unit disk.
8. You should know the definition of convergence for series. You should know what it means for a series to converge absolutely.
9. You should know geometric series thoroughly. You should know when it converges and why, when it doesn't converge and why. You should know how to produce the sum.
10. You should know how to investigate convergence and all of the associated results we discussed in the textbook: The comparison test and the ratio test.
11. For a sequence of functions,  $\{F_n(z)\}$ , you should know what it means for them to converge (pointwise) on a set and what it means for them to converge uniformly on a set.
12. We talked a lot about the things that are preserved under uniform convergence. For example, we proved that if  $\{F_n(z)\}$  converges uniformly to  $F$  on a domain  $\mathcal{D}$  and all of the functions  $F_n$  are continuous, then  $F(z)$  must also be continuous on  $\mathcal{D}$ . We also proved that, if  $\{F_n(z)\}$  converges uniformly to  $F$  on  $\mathcal{D}$  and  $\Gamma$  is any contour in  $\mathcal{D}$ , then

$$\lim_{n \rightarrow \infty} \int_{\Gamma} F_n(z) dz = \int_{\Gamma} F(z) dz.$$

You should know these results and how to prove them.

13. You should be able to translate both of the above results to the context of series. For example, you should be able to prove that, if a series  $\sum_{n=1}^{\infty} f_n(z)$  converges uniformly on a domain  $\mathcal{D}$ , then, for any contour  $\Gamma$  in  $\mathcal{D}$ ,

$$\int_{\Gamma} \sum_{n=1}^{\infty} f_n(z) dz = \sum_{n=1}^{\infty} \int_{\Gamma} f_n(z) dz,$$

in particular, that the latter numerical series converges.

14. You should know Taylor's theorem for analytic functions and how it is proved (with the proof's basic ingredients being Cauchy's theorem and geometric series).
15. You should know all of the general facts about power series. This includes, the radius of convergence, algebraic operators, where such a series converges, where it uniformly converges, etc. You should also know our result connecting general power series to series: If  $f(z)$  is given by a convergent power series about a point  $z_0$  with a radius of convergence  $R > 0$ , then  $f(z)$  is analytic and the defining series is the Taylor series for  $f$  (this is Theorem 10+11 on Page 257).
16. You should know the statement Theorem 14 and have a good idea of how that theorem is proved.

**Some 'Do Not Turn In' Exercises**

1. Do Exercises 1, 3, 5, 7, 9, and 15 in Section 5.3
2. Do Exercises 1, 3, 4, 7, and 11 in Section 5.5.