1 Final Location and Time

Our final exam is at the following place/time:

Malott 406 – 7:00PM – 12/02/2021

2 Office Hours

In preparation for the final exam, I will hold office hours at the following times:

1. Friday, 1-3PM
2. Sunday, 3-5PM

I can also be available other times (either in person or via Zoom). If you'd like to talk to me and cannot make the above listed times, please send me an email and we can set something up.

3 List of Topics

As promised, below is an itemized list of topics we've covered in Math 4220 since the second prelim. Approximately 34% of the Final exam will correspond to this post-prelim-2 material. The remaining 66% of the exam will be based on material covered before the prelim. Thus, please use this topics list in conjunction with the two previous prelim topics list as aides as you prepare for the final exam. As I stated before, please note that I consider the homework exercises and the examples I've covered in lecture to be the best source of practice problems. If you know how to approach each problem, are able to work quickly and accurately, and understand the theory and methodology by which you have obtained a solution, you should perform well on the exam.

1. You should know the Laurent series theorem (Theorem 14 on Page 269), how it is proved and how it is applied in various settings.
2. Given a relatively simple function, you should be able to compute its Laurent series at a specific point.
3. You should know the basics of Fourier series as they followed from the Laurent series theorem (at least in looked at the restriction to the circle of an analytic function). Some of this material was outlined in Homework 9.
4. For a 2π-periodic function f, you should be able to compute its Fourier coefficients $\widehat{f}(n)$ and write down its Fourier series.
5. Insofar as I discussed it in class, you should understand generally when a function's Fourier series converges to the function.
6. For a function f and a point z_0 in its domain of analyticity, you should know what it means for z_0 to be a zero of order m for f.
7. You should know the characterizing condition of zeros given in Theorem 16 on Page 278 (and how it is proved).
8. For a function f having an isolated singularity at z_0, you should know what it means for z_0 to be a removable singularity, a pole of order m and an essential singularity. That is, you should know Definition 8 precisely.
9. You should know the consequences of z_0 being a removable singularity (see Lemma 5)
10. You should know the characterization of poles given in Lemma 7 (this is usually how we determine when a function has a pole of order m). You should know its proof.
11. You should understand/appreciate the stark difference in behaviors of a function f which has a pole of order m at z_0 (See Lemma 6) and an essential singularity at z_0 (Picard's theorem).

12. For a function f and z_0, an isolated singularity of f, you should know how the residue $\text{Res}(f; z_0)$ is defined.

13. If you are able to find a Laurent series for a function (fairly easily), you should be able to immediately say what the residue is.

14. You should know Theorem 1 on Page 310 and how it is proved. Note: One almost always uses this theorem to compute residues.

15. You should know Cauchy's Residue theorem.

16. You should know the utility of residues and Cauchy's Residue Theorem in computing integrals. To this end:

    (a) You should be able to use Cauchy's residue theorem to compute various trigonometric integrals appearing in Section 6.2.

    (b) Similarly, you should be able to use Cauchy's residue theorem to compute various integrals (including Cauchy's principal value) on the line $\mathbb{R} = (-\infty, \infty)$ as presented in Section 6.3.

    (c) In particular, you should be prepared to work an integral like I did in Class on December 7. That is, I used Cauchy's Residue Theorem to compute

    $$p.v. \int_{-\infty}^{\infty} \frac{1}{x^4 + 4}\, dx.$$

    (d) What's important about these ideas is that various "real integrals" which show up in applied mathematics (many of which, a priori, have nothing to do with complex analysis) can be computed via the techniques of complex analysis.

17. Don't worry too much about the material I covered in Section 8.2. You should have the general idea, but I won't ask about things very precisely on the final.