1. Introduction

My research combines contact and symplectic geometry with low-dimensional topology to study dynamics in two, three, and four dimensions. The invariant I use, *embedded contact homology (ECH)*, is powerful but difficult to compute. There are two filtrations on ECH that induce spectral invariants: the *ECH capacities* (§1.1), derived from the *symplectic action*, are used to obstruct symplectic embeddings, while *knot-filtered ECH* (§2), derived from linking number, is used to study surface dynamics and the knotting of Reeb orbits. Computing either requires understanding the ECH chain complex, so a priori cannot pass through either of the isomorphic theories of Seiberg-Witten or Heegaard Floer homologies.

My research leverages low-dimensional topology and toric geometry to construct, compute, and apply ECH spectral invariants to geometric problems. I follow two major directions. Firstly, I work to extend the foundations of knot-filtered ECH (§2.2), along the way providing computations of this relatively new invariant (§2.1). Secondly, I am interested in codifying the relationship between ECH cobordism maps and other symplectic embedding obstructions while computing new embedding capacity functions (§3.3) and solving a case of a conjecture of Cristofaro-Gardiner–Holm–Mandini–Pires classifying symplectic embeddings [10] (§3.2, 3.3).

1.1. **Central definitions.** We use Y to denote a smooth, closed, oriented three-manifold. A *contact form* is a one-form λ on Y with $\lambda \wedge d\lambda > 0$. If $Y \subset \mathbb{C}^2$ then often $\lambda_0|_Y$ is a contact form, where $\lambda_0 = \frac{1}{2}\sum_{i=1}^{2} x_i dy_i - y_i dx_i$. Contact forms induce *Reeb vector fields*: smooth nonzero vector fields on Y determined by $d\lambda(R, \cdot) = 0$ and $\lambda(R) = 1$. Closed integral curves of the Reeb vector field, i.e. $\gamma : R/T\mathbb{R} \to Y$ with $\gamma'(t) = R_{\gamma(t)}$, modulo reparameterization, are called *Reeb orbits*, which are *embedded* when they are injective. The *(symplectic) action* of a Reeb orbit γ is $\int_\gamma \lambda$.

My main tool is ECH, a Floer homology theory developed by Hutchings [18] and generated by certain finite sets of Reeb orbits. We denote by J an *almost complex structure* on $\mathbb{R} \times Y$, which is a complex structure on $T(\mathbb{R} \times Y)$ not necessarily arising from holomorphic coordinate charts. A *J-holomorphic* (or *pseudoholomorphic*) curve is a map from a (perhaps punctured or disconnected) Riemann surface into $\mathbb{R} \times Y$ satisfying the nonlinear Cauchy-Riemann equation determined by J. When α and β are ECH generators, the coefficient of β in $\partial\alpha$ is a count of J-holomorphic curves asymptotic to cylinders over the orbits in α and β as the $\mathbb{R}$-coordinate goes to $\pm\infty$. At the homology level, ECH depends only on $(Y, \ker\lambda)$ and not J [29]; thus we denote the ECH of (Y, λ) by $ECH_*(Y, \ker\lambda)$. ECH was used to prove the three-dimensional Weinstein conjecture [28], characterize symplectic embeddings [24], and satisfies a Weyl law [11]. There are two key *spectral invariants* in ECH, ECH capacities and knot-filtered ECH, derived from *filtrations*, i.e. functions on the ECH chain complex which decrease with respect to the differential. To compute either spectral invariant, we need to compute the entire ECH chain complex directly from λ.

A *symplectic form* on a smooth $2n$-manifold X is a smooth two-form ω with $d\omega = 0, \omega^{\wedge n} \neq 0$. If $\omega = d\lambda$ and $\lambda|_{\partial X}$ is a contact form, the *ECH capacities* of (X, ω) are

$$0 = c_0(X, \omega) < c_1(X, \omega) \leq c_2(X, \omega) \leq \cdots \leq \infty,$$

where $c_k(X, \omega)$ measures the least symplectic action necessary to realize certain classes in $ECH_{2k}(\partial X, \lambda)$ by homologically essential cycles of Reeb orbits for a contact form primitive of ω [17].

ECH capacities obstruct *symplectic embeddings* from (X, ω) to (X', ω'), which are smooth embeddings $\phi : X \to X'$ with $\phi^*\omega' = \omega$. We use the notation $X \overset{s}{\hookrightarrow} X'$ when ω, ω' are understood. The isomorphism between ECH and Seiberg-Witten Floer homology [28, 20] implies

$$X \overset{s}{\hookrightarrow} X' \Rightarrow c_k(X, \omega) \leq c_k(X', \omega') \, \forall k. \tag{1.1}$$

2. KNOT-FILTERED ECH

In my 2019 thesis, I investigated *knot-filtered ECH*, a spectral invariant which measures the linking of ECH generators with a fixed Reeb orbit. It was introduced by Hutchings in [19] for Y with $H_1(Y) = 0$. We denote by $ECH^\ell(Y, \ker\lambda, B, \theta)$, where B is a Reeb orbit, $\ell \in \mathbb{R}$, and θ is a parameter depending on $d\lambda$ near B, the homology of the subcomplex generated by orbits of linking number at most ℓ with B. knot-filtered ECH likely exists for general Y, and in my thesis I defined it when $b_1(Y) = 0$ and proved invariance of λ and J:

Theorem 2.1 (W '19 [31])**.** *If $b_1(Y) = 0$, $ECH^\ell(Y, \ker\lambda, B, \theta)$ is defined and independent of λ, J.*

I applied Thm. 2.1 to lens spaces $Y = L(p, p-1)$ with B either component of the image of the Hopf link under the quotient $L(p, p-1) = S^3/\mathbb{Z}_p$, computed their knot-filtered ECH, and proved:

Theorem 2.2 (W '19 [31])**.** *Such Y have a Reeb orbit γ whose action has an upper bound given by its linking numbers with the components of image of the Hopf link and $\int_Y \lambda \wedge d\lambda, p$, and θ.*

Bechara Senior–Hryniewicz–Salomão have generalized Thm. 2.2 to generic (Y, λ) [2].

In my thesis [31] I used Thm. 2.2 to prove a quantitative smooth version of Franks' theorem [14] that area-preserving homeomorphisms of the annulus have two or infinitely many periodic orbits:

Theorem 2.3 (W '19 [31])**.** *If ψ is a symplectomorphism of the annulus, a rotation near the boundary, and its "Calabi invariant" (an average measure of rotation) is less than its larger boundary rotation number, then ψ has infinitely many periodic orbits.*

I proved Thm. 2.3 by embedding the annulus transversally to the Reeb vector field of a contact $L(p, p-1)$ as in Thm. 2.2, realizing ψ as the return map of the Reeb flow from the annulus to itself. This method was inspired by Hofer's "global surfaces of section" [16] and the case of the disk in [19]; see also [1] for a proof of the smooth Hamiltonian closing lemma using similar methods.

To date, direct computations of ECH require an effective torus action on Y preserving λ. In order to analyze more complex (Y, λ), it is essential that we understand the ECH chain complex of non-toric (Y, λ). My work on the ECH of circle bundles, joint with Jo Nelson, is the first step.

Theorem 2.4 (NW [27])**.** *Let Y be an S^1 bundle over a surface with a contact form λ whose Reeb flow is the fiber action. Then $ECH_*(Y, \ker\lambda)$ is the exterior algebra of the homology of the surface.*

The contact forms studied in Thm. 2.4 exist on all circle bundles [5]. Thm. 2.4 first appeared in a $\mathbb{Z}_2$-graded form in the unpublished thesis of Farris [12]; Nelson and I completed the J-holomorphic curve analysis and proved the $\mathbb{Z}$-graded version. Our work was used by Ferreira-Ramos to compute the ECH capacities of some disk bundles in [13].

2.1. New computations.

I have computed new examples of knot-filtered ECH:

Example 2.5 (NW [26])**.** Denote the positive trefoil by $T_{3,2}$. If $\theta \approx 6$, then $ECH^\ell_*(S^3, \ker\lambda_0|_{S^3}, T_{3,2}, \theta)$ is nonzero if and only if $\ell > c_k(E(3,2))$, where $E(3,2)$ is the "ellipsoid" defined in §3.

Understanding the knot-filtered of other λ will require new computational methods. There is a Seifert fibration of S^3 with $T_{\pm 3,2}$ as a regular fiber, inducing a contact form with the S^1 action as the Reeb flow. By viewing the Reeb flow as an S^1-bundle over an orbifold, Nelson and I will generalize Thm. 2.4 to compute the knot-filtered ECH of the left-handed trefoil:

Project 2.6 (NW [26])**.** *Compute $ECH^\ell_*(S^3, \ker\lambda_0|_{S^3}, T_{-3,2}, \theta)$ for $\theta \approx -6$.*

Although the Reeb vector fields in Ex. 2.5 and Proj. 2.6 are isomorphic as Seifert fibrations, their contact structures are far from contactomorphic: one is tight and the other overtwisted. Project 2.6 will thus reveal more of the dependence of knot-filtered ECH on the contact structure.

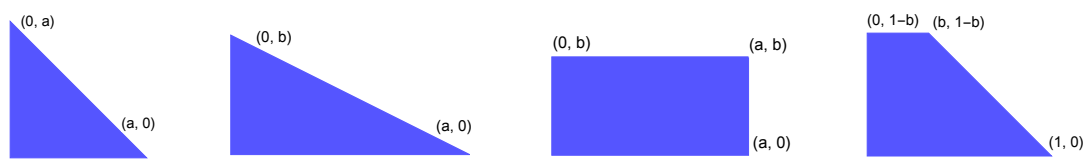
2.2. **Foundations and relations to Heegaard Floer theory.** In order to study the linking properties of Reeb orbits for general Y – in particular, circle bundles on surfaces, whose ECH chain complex we analyzed to prove Thm. 2.4 – I am extending the foundations of knot-filtered ECH:

Project 2.7 (Weiler)**.** *Prove knot-filtered ECH is defined for all Y, and independent of λ, J.*

Knot (Heegaard) Floer homology contains two invariants, τ and Υ, whose constructions are reminiscent of that of knot-filtered ECH. In particular, τ is similar to the smallest value ℓ for which $ECH_0^\ell(S^3, \ker\lambda, K, \theta)$ is nonzero, although τ does not depend on the parameter θ recording the contact form's differential $d\lambda$ near K. The invariant Υ extends τ using a U-filtration which also appears in ECH. Therefore, I will investigate whether τ and Υ can be computed from knot-filtered ECH, perhaps by fixing or limiting over the parameter θ.

3. Symplectic embeddings and the ECH spectrum

My research program also investigates symplectic embedding problems, for which ECH invariants have proved very useful (see [7]). Symplectic forms induce volumes: we define $\mathrm{vol}(X,\omega) := \int_X \omega^{\wedge n}$, and $(X,\omega) \overset{s}{\hookrightarrow} (X',\omega')$ only if $\mathrm{vol}(X,\omega) \leq \mathrm{vol}(X',\omega')$. Therefore a central theme in symplectic geometry is discerning how it is "rigid," i.e. similar to complex geometry, versus "flexible," i.e. similar to volume-preserving geometry. Symplectic embeddings are at the heart of this rigidity-flexibility dichotomy. With his proof of "Gromov nonsqueezing," Gromov proved that $B(r) \overset{s}{\hookrightarrow} Z(R) \Rightarrow r \leq R$, where $B(r)$ is the finite-volume ball (Fig. 1 (a)) and $Z(R)$ is the infinite-volume cylinder $\{z \in \mathbb{C}^n | \pi|z_1|^2 < R\}$ with the symplectic form ω_0 (Def. 3.1) [15]. Nonsqueezing is a rigid phenomenon because embeddings are obstructed by just one local parameter, the area of each manifold's intersection with the $\mathbb{C} \times \{0\}$ axis. Yet there are also flexible cases, meaning that volume and topology are the only obstruction to embedding: [4], [25]. My work investigates the exact boundary between rigidity and flexibility.



(a) *Ball* $B(a)$ (b) *Ellipsoid* $E(a,b)$ (c) *Polydisk* $P(a,b)$ (d) X_b

Figure 1. Several Ω determining toric domains (Def. 3.1), with X_Ω named. All are convex; ellipsoids are also concave.

Definition 3.1. For Ω a region in the first quadrant of $\mathbb{R}^2$, the *toric domain* (X_Ω, ω_0) is

$$X_\Omega := \{(z_1,z_2) \in \mathbb{C}^2 | (\pi|z_1|^2, \pi|z_2|^2) \in \Omega\},\ \omega_0 = \textstyle\sum_{i=1}^2 dx_i \wedge dy_i|_{X_\Omega}.$$

If Ω is the region under the graph of a concave (resp., convex) function, X_Ω is *convex* (resp., *concave*), and $\lambda_0|_Y$ is a contact form on ∂X with $d\lambda_0 = \omega_0$. See Fig. 1 for key examples.

3.1. **Past work.** Even the question of whether $E(a,b) \overset{s}{\hookrightarrow} E(c,d)$ is very difficult to answer [23]. In 2020 work with Bertozzi, Holm, Maw, Mwakyoma, McDuff, and Pires, I analyzed symplectic embeddings into a continuous range of toric domains, discovering delicate and complex behaviors. Because we always have $E(a,1) \overset{s}{\hookrightarrow} cX$ if $c >> 0$, we study embeddings of ellipsoids into X via the *ellipsoid embedding function*

$$c_X(a) := \inf\{c | (E(a,1),\omega_0) \overset{s}{\hookrightarrow} (X, c\omega_0)\},$$

first investigated by McDuff and Schlenk [25]. They found $c_{B(1)}$ contained an *infinite staircase*, i.e., $c_{B(1)}$ is smooth or piecewise linear, and contains infinitely many nonsmooth points whose a-coordinates accumulate at the golden ratio to the fourth. Ellipsoid embedding functions always satisfy the lower bound

$$c_X(a) \geq \mathrm{vol}_X(a), \tag{3.1}$$

where $\mathrm{vol}_X(a)$ is the smallest value of c so that $\mathrm{vol}(X, c\omega_0) = \mathrm{vol}(E(1,a), \omega_0)$. The numerics of this staircase describe regions of alternately rigid and flexible embeddings: when (3.1) is a strict inequality, all embeddings $(E(1,a), \omega_0) \overset{s}{\hookrightarrow} (X, c\omega_0)$ are rigid, whereas the embedding $(E(1,a), \omega_0) \overset{s}{\hookrightarrow} (X, \mathrm{vol}_X(a)\omega_0)$ is flexible, filling all available volume.

Previous researchers have investigated c_X for other convex X, discovering some with infinite staircases and some without: [30, 10, 8]. Although there are shared techniques, small adjustments to Ω can drastically change the problem. With Bertozzi, Holm, Maw, Mwakyoma, McDuff, and Pires, I studied c_{H_b}, where H_b is the complex projective plane blown up once with weight $b \in [0,1)$. We have $c_{H_b} = c_{X_b}$ (Fig. 1 (d)) by [10, Thm. 1.3], allowing us to use ECH tools to study c_{H_b}.

Theorem 3.2 (BHMMMPW [3])**.** *c_{H_b} has an infinite staircase for six infinite families of b values.*

We identified the values of b in Thm. 3.2 were with computer programs approximating c_{H_b} via ECH capacities. McDuff proved (1.1) is an equivalence for $X = E(a,b), X' = E(c,d)$ [24], and Cristofaro-Gardiner generalized her work to show (1.1) is an equivalence if $X = X_\Omega$ is concave and $X' = X_{\Omega'}$ is convex [7]. The ECH capacities of convex and concave X_Ω are combinatorially computable, but using (1.1) to compute c_{X_Ω} exactly is generally impossible, because it requires understanding an infinite sequence of capacities. The best we can do is approximate c_{X_Ω}, and this is still difficult, because the new infinite staircases are only visible if tens of thousands of ECH capacities have been computed. Previously, researchers had only been able to compute hundreds or thousands of capacities in any reasonable time ([10, 30]), leaving the staircases of Thm. 3.2 invisible. Our work leveraged direct computations of the ECH of $\partial(X_b)$ to optimize the algorithms for $c_k(X_b)$, illuminating our new infinite staircases, as well as some of those later investigated by Magill-McDuff (see next paragraph) and Magill-McDuff-Weiler (Thm. 3.3). Another new feature of our work in [3] is the discovery of descending infinite staircases: all previously known infinite staircases accumulated from below.

Thm. 3.2 was recently extended to parallel Usher's results on embeddings into polydisks [30] and include descending staircases (which Usher did not discuss, but which conjecturally also appear in the polydisk case): in [21], Magill–McDuff found a bi-infinite family of values of b for which c_{H_b} has an infinite staircase, extending the original six. In recent work [22] with Magill and McDuff, I have further extended this bi-infinite family, uncovering uncountably many new staircases with no parallel to Usher's:

Theorem 3.3 (Magill–McDuff–Weiler [22])**.** *There is an open dense subset $B \subset [6,8]$ such that:*

- *(i) $[6,8] \setminus B$ is homeomorphic to the Cantor set.*
- *(ii) For each $a \in [6,8] \setminus B$, there is an ascending or descending infinite staircase, or both, with accumulation point $(a, \mathrm{vol}_X(a))$.*
- *(iii) The generalizations of [21] extend (ii) to determine for all $b \in [0,1)$ whether c_{H_b} has an infinite staircase, except perhaps one countable family with descending infinite staircases.*

We expect the potential descending staircases in *(iii)* do not exist: see §3.2.

3.2. **Cristofaro-Gardiner–Holm–Mandini–Pires conjecture.** In [10], Cristofaro-Gardiner–Holm–Mandini–Pires conjecture that up to scale there are only 12 convex X_Ω where Ω is a polygon with corners in $\mathbb{Q}^2$ for which c_{X_Ω} has an infinite staircase. They also showed that the a-coordinate

of any infinite staircase c_{H_b} with rational b must be either rational or a quadratic irrational. With Magill and McDuff, I have proved part of this conjecture:

Theorem 3.4 (Magill–McDuff–Weiler [22])**.** *The only values of b where c_{H_b} has an ascending infinite staircase and an accumulation point with rational a-coordinate are $b = 0, \frac{1}{3}$.*

In future work we plan to extend Thm. 3.4 to rule out descending infinite staircases (Thm. 3.3 *(iii)*) as well as quadratic irrational a-coordinates of accumulation points, thus proving the conjecture for X_b (and thus H_b). We hope our work will provide a blueprint for proving the conjecture more generally (ellipsoids were treated in [6]; our methods should extend to polydisks).

3.3. **ECH cobordism maps and embedding obstructions.** When Ω is the region under the graph of a concave piecewise linear function with rational edge slopes, c_{X_Ω} can be computed as the supremum over an infinite family of either ratios of ECH capacities (as described above), or over an infinite family of obstruction functions determined by symplectically embedded spheres in a blowup of the symplectic toric manifold obtained by collapsing a S^1-fibration of $\partial(\Omega)$; this method was pioneered by [25] and extended in [10].

The heuristic relationship between the two perspectives is the following. If $\varphi : (X, \omega) \overset{s}{\hookrightarrow} (X', \omega')$, then $X' \setminus \varphi(X)$ is a symplectic cobordism. A natural way to define cobordism maps from the ECH of $\partial X'$ to that of $\partial(\varphi X)$ (equivalently ∂X, since ECH is a diffeomorphism invariant) is via counting J-holomorphic curves in $(X' \setminus \varphi(X), \omega')$. ECH capacities of X and X' would differ by at least the symplectic areas of these curves, and in nice cases, by exactly these areas. When X, X' are toric domains with rational edge slopes, $\partial X, \partial(\varphi X)$ can be collapsed along the S^1-fibration by the Reeb vector fields of their natural contact forms. Any J-holomorphic curves collapse to symplectically embedded surfaces, and if they are spheres and X is an ellipsoid, the obstruction to $c_{X'}$ from the cobordism map equals that from the image of the collapsed J-holomorphic curve. This perspective was initiated in [9].

Experimentally, the above heuristic holds. In all cases checked, for certain intervals of a values the lower bound on $c_{X'}$ derived from the kth ECH capacity equals the lower bound on $c_{X'}$ derived from a symplectically embedded sphere in the correct second homology class to arise from collapsing the boundary of a curve counted by the ECH cobordism map in index $2k$. In [3, Lem. 92] we proved one direction of this correspondence when $X = H_b$.

I plan to explore this potential equivalence further. There are several results in the literature which have only been proved using either ECH capacities or symplectically embedded spheres, while using the other method would extend the result. Most significant is a Cristofaro-Gardiner–Holm–Mandini–Pires result providing the coordinates of the accumulation point of an infinite staircase ([10, Thm. 1.11]), which is crucial to all the work described in §3.1. It was proved using symplectically embedded spheres, and inherent in that method is that the proof only works for restricted Ω. The analogous proof with ECH capacities would include many Ω whose c_{X_Ω} we currently know nothing about, for example $X_\Omega = E(b, 1)$ with b irrational.

On the other hand, [3, Thm. 94], which proves $c_{H_{\frac{1}{5}}}$ has no infinite staircase, was proved using ECH capacities using a method that is hard to generalize to other rational values of b where c_{H_b} is unobstructed at the accumulation point but has no infinite staircase (these form the countable family in Thm. 3.3 *(iii)*). Translating this proof to symplectically embedded spheres may make it more generalizable, and will aid in proving the Cristofaro-Gardiner–Holm–Mandini–Pires conjecture.

Project 3.5. *(1) Prove the heuristic equivalence above. That is, prove that the ECH cobordism map actually counts genus zero J-holomorphic curves in the correct homology classes, as predicted by* [25, 10]*, and extending* [9, Prop. 5]*. This generalizes* [3, Lem. 92]*.*

(2) Compute $c_{E(b,1)}$ for some irrational b (this seems to only be possible using ECH capacities).

(3) *Prove the Cristofaro-Gardiner–Holm–Mandini–Pires conjecture, by translating the proof of* [3, Thm. 94] *to symplectically embedded spheres.*

References

[1] Masayuki Asaoka and Kei Irie. A C^∞ closing lemma for Hamiltonian diffeomorphisms of closed surfaces. *Geom. Funct. Anal.*, 26(5):1245–1254, 2016.

[2] David Bechara Senior, Umberto Hryniewicz, and Pedro A. S. Salomao. On the relation between action and linking. *arXiv:2006.06266*.

[3] Maria Bertozzi, Tara S. Holm, Emily Maw, Dusa McDuff, Grace Mwakyoma, Ana Rita Pires, and Morgan Weiler. Infinite staircases for Hirzebruch surfaces. *submitted. arXiv:2010.08567*.

[4] Paul Biran. Symplectic packing in dimension 4. *Geom. Funct. Anal.*, 7(3):420–437, 1997.

[5] William M. Boothby and Hsien Chung Wang. On contact manifolds. *Ann. of Math. (2)*, 68(3):721–734, 1958.

[6] Daniel Cristofaro-Gardiner. Special eccentricities of rational four-dimensional ellipsoids. *arXiv:2004.13647*.

[7] Daniel Cristofaro-Gardiner. Symplectic embeddings from concave toric domains into convex ones. *J. Diff. Geom.*, 112(2):199–232, 2019.

[8] Daniel Cristofaro-Gardiner, David Frenkel, and Felix Schlenk. Symplectic embeddings of four-dimensional ellipsoids into integral polydiscs. *Algebr. Geom. Topol.*, 17(2):1189–1260, 2017.

[9] Daniel Cristofaro-Gardiner and Richard Hind. Symplectic embeddings of products. *Comment. Math. Helv.*, 93(1):1–32, 2018.

[10] Daniel Cristofaro-Gardiner, Tara S. Holm, Alessia Mandini, and Ana Rita Pires. On infinite staircases in toric symplectic four-manifolds. *arXiv:2004.13062*.

[11] Daniel Cristofaro-Gardiner, Michael Hutchings, and Vinicius G. B. Ramos. The asymptotics of ECH capacities. *Invent. Math.*, 199(1):187–214, 2015.

[12] David Farris. *The embedded contact homology of nontrivial circle bundles over Riemann surfaces*. PhD thesis, University of California, Berkeley, 2011.

[13] Brayan Ferreira and Vinicius G. B. Ramos. Symplectic embeddings into disk cotangent bundles, 2021. arXiv:2110.02312.

[14] John Franks. Geodesics on S^2 and periodic points of annulus homeomorphisms. *Invent. Math.*, 108(2):403–418, 1992.

[15] Mikhail Gromov. Pseudo holomorphic curves in symplectic manifolds. *Invent. Math.*, 82(2):307–347, 1985.

[16] Helmut Hofer. Pseudoholomorphic curves in symplectizations with applications to the Weinstein conjecture in dimension three. *Invent. Math.*, 114:515–563, 1993.

[17] Michael Hutchings. Quantitative embedded contact homology. *J. Diff. Geom.*, 88(2):231–266, 2011.

[18] Michael Hutchings. Lecture notes on embedded contact homology. In *Contact and symplectic topology*, volume 26 of *Bolyai Soc. Math. Stud.*, pages 389–484. János Bolyai Math. Soc., Budapest, 2014.

[19] Michael Hutchings. Mean action and the Calabi invariant. *J. Mod. Dyn.*, 10:511–539, 2016.

[20] Michael Hutchings and Clifford Henry Taubes. Proof of the Arnold chord conjecture in three dimensions, II. *Geom. Topol.*, 17(5):2601–2688, 2013.

[21] Nicki Magill and Dusa McDuff. Staircase symmetries in Hirzebruch surfaces, 2021. arXiv:2106.09143.

[22] Nicki Magill, Dusa McDuff, and Morgan Weiler. Staircase patterns in Hirzebruch surfaces, 2021. in progress.

[23] Dusa McDuff. Symplectic embeddings of 4-dimensional ellipsoids. *J. Topol.*, 2(1):1–22, 2009.

[24] Dusa McDuff. The Hofer conjecture on embedding symplectic ellipsoids. *J. Diff. Geom.*, 88(3):519–532, 2011.

[25] Dusa McDuff and Felix Schlenk. The embedding capacity of 4-dimensional symplectic ellipsoids. *Ann. of Math. (2)*, 175(3):1191–1282, 2012.

[26] Jo Nelson and Morgan Weiler. In progress.

[27] Jo Nelson and Morgan Weiler. Embedded contact homology of prequantization bundles. *submitted. arXiv:2007.13883*.

[28] Clifford Henry Taubes. The Seiberg-Witten equations and the Weinstein conjecture. *Geom. Topol.*, 11:2117–2202, 2007.

[29] Clifford Henry Taubes. Embedded contact homology and Seiberg-Witten Floer cohomology I. *Geom. Topol.*, 14(5):2497–2581, 2010.

[30] Michael Usher. Infinite staircases in the symplectic embedding problem for four-dimensional ellipsoids into polydisks. *Algebr. Geom. Topol.*, 19(4):1935–2022, 2019.

[31] Morgan Weiler. Mean action of periodic orbits of area-preserving annulus diffeomorphisms. doi:10.1142/S1793525320500363, 2019. Accepted to the *J. Topol. Anal.* Oct. 29, 2019.