

Typos from Fortney

Please let me know if you find any typos and I will add them here.

Page 25 In the definition of the a_{ij} at the bottom of the page, the last offset equation should read

$$Df(x) \cdot e_j = \sum_{i=1}^m a_{ij} \tilde{e}_i = \begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$$

Page 26 In the last three offset equations on the page, each x_{i_0} should be replaced with x_{j_0} .

Page 45 In Definition 2.3.2, the first input $t_0 + ta$ of f should be $x_0 + ta$.

Pages 50-52 The notation switches between $\left. \frac{\partial f}{\partial x_i} \right|_p$ and $\left. \frac{\partial f}{\partial x_i} \right|_{p_i}$; it should always be the former. In particular, in the computation at the bottom of page 50, each $\frac{\partial f}{\partial x_i}$ should be evaluated at $(x_1(0), \dots, x_n(0))$ (in the third line) or at p (in the fourth line).

Page 67 In Question 2.31, both ω and α indicate the same one-form.

Page 67 In Question 2.33, it isn't possible to evaluate df_p for some of the f s of Question 2.32 (specifically (e), (g), (h), and (i), as those involve roots and \ln , which cannot be applied to negative numbers). It also isn't possible to evaluate $df[v_p]$ when $v_p \in T_p \mathbb{R}^3$ for (a)-(c) of Question 2.32, because in those cases $f : \mathbb{R}^2 \rightarrow \mathbb{R}$.

Page 86 All discussion of counting the number of two-dimensional subspaces refers to the two-dimensional subspaces spanned by combinations of the standard basis vectors.

Page 91 In the first equation, dx^2 should be dx_2 .

Page 91 In the formula for the wedge product of two arbitrary forms, the sum on the right hand side is over all $I \in \mathcal{I}_{k,n}$ and $J \in \mathcal{I}_{\ell,n}$.

Page 96 In the first line after the equations, "indice" should be "index."

Page 99 In the third line of the computation of $\iota_v(\alpha \wedge \beta)$ in the middle of the page, dx_p should be dx_k .

Page 99 The last two lines on the page are missing some minus signs. They should read, after the equals sign:

$$\begin{aligned} &= \sum_{i=1}^{k-1} (-1)^{i-1} dx_i(v) \sum_{j=i+1}^k (-1)^{(j-1)-1} dx_j(u) (dx_1 \wedge \cdots \widehat{dx_i} \cdots \widehat{dx_j} \cdots \wedge dx_k) \\ &\quad + \sum_{i=2}^k (-1)^{i-1} dx_i(v) \sum_{j=1}^{i-1} (-1)^{j-1} dx_j(u) (dx_1 \wedge \cdots \widehat{dx_j} \cdots \widehat{dx_i} \cdots \wedge dx_k) \end{aligned}$$

Page 176 The descriptor of Figure 5.33 should read “As an example, a generic one-form α on \mathbb{R}^2 is shown in Figure 5.33.”

Page 147 In Question 4.17, you should be asked to show

$$d(\phi \wedge \psi) = d\phi \wedge \psi - \phi \wedge d\psi$$

Page 149 Not exactly a typo, but a note: the result of Question 4.33 holds for \mathbb{R}^n , not just \mathbb{R}^2 .

Page 228 In Question 6.27, α is a two-form, not a one-form.

Integration and orientations The formula

$$\int_R f dx_1 \wedge \cdots \wedge dx_n = \int_{\phi(R)} (\phi^{-1})^* (f dx_1 \wedge \cdots \wedge dx_n)$$

does not work in general – it only works if ϕ “preserves orientation.” If ϕ “reverses orientation,” then we have to add a minus sign:

$$\int_R f dx_1 \wedge \cdots \wedge dx_n = - \int_{\phi(R)} (\phi^{-1})^* (f dx_1 \wedge \cdots \wedge dx_n).$$

Similarly, when integrating a k -form over a k -manifold,

$$\int_{\Sigma_k} \alpha = \int_{\phi(\Sigma_k)} (\phi^{-1})^* \alpha$$

only if ϕ preserves orientation (this requires choosing an orientation for Σ_k !), and

$$\int_{\Sigma_k} \alpha = - \int_{\phi(\Sigma_k)} (\phi^{-1})^* \alpha$$

if ϕ reverses orientation.

Page 235 In the first line of the displayed equations at the bottom of the page, the integral with respect to x should be from zero to $y + 2$, not from $y + 2$ to zero. You can check this because the answer he gets for the integral is negative, while the function he’s integrating is positive over the region R of integration.

Page 238-9 The formula $\int_{\phi(R)} f du \wedge dv = \iint_{\phi(R)} f dudv$ only works if $du \wedge dv$ is a properly oriented volume form. In this case it’s not, which we know because $(\phi^{-1})^* du \wedge dv = -\frac{1}{2} dx \wedge dy$. So we can’t compute

$$\int_{\phi(R)} f du \wedge dv = \iint_{\phi(R)} f dudv;$$

instead we have to compute

$$- \int_{\phi(R)} f dudv.$$

Page 243 There is a sign error in this parameterization; the correct integrand is the negative of Fortney's.

Page 255-257 Throughout these problems Fortney writes ϕ where our convention is to use ϕ^{-1} for parameterizations.

§8.3 Throughout §8.3, we see $f_{i_1 \dots i_k}$ rather than $f_{i_1 \dots i_k}$. Fortney is also not consistent with using y variables for \mathbb{R}^{n+1} and x variables for \mathbb{R}^n .

Page 271 In the last line of the first displayed equations, only (y_1, \dots, y_n) should be the inputs to $f_{i_1 \dots i_k}$, not (y_1, \dots, y_{n+1}) .

Page 253 In Example Two (b), the parameterization should be

$$(\theta, \phi) \mapsto (\sin \phi \cos \theta, \sin \phi \sin \theta, \cos \phi).$$

Page 274 In Question 8.9, the sum defining the general k -form α should be over $J \in \mathcal{I}_{k,n}$, not \sum_i^{n+1} .