1. Introduction: pseudoholomorphic curves in low dimensions

My research is low-dimensional contact and symplectic geometry with applications to dynamics. Contact and symplectic geometry provide the framework for classical mechanics, modeling planetary motion and electromagnetism, and connect many fields of mathematics. My research goals are:

- expanding the topological and geometric settings in which we can compute the pseudoholomorphic curve invariants central to modern contact and symplectic geometry,
- optimizing these computations to discover new phenomena, and
- applying these computations to 4D symplectic embedding problems, the topology of Reeb orbits in 3D, the 4D Viterbo conjecture, and 2D dynamics.

My work has connections with complex algebraic geomety, nonlinear PDE, low-dimensional topology, knot theory, number theory, Ehrhart theory, and topological quantum field theory; some projects require extensive computer programming. With my coauthors, I have worked on:

- *Symplectic embeddings §2:* I proved a foundational case of a conjecture of Cristofaro-Gardiner–Holm–Mandini–Pires arising from Gromov nonsqueezing in [MMW22, MPW].
- *New computations of ECH §3:* I computed the embedded contact homology of all contact forms with free circle actions in [NW20] (used by Ferreira-Ramos in [FR21]), and of a family of contact forms with effective actions in [NW].
- *Knot filtration §4:* In [Wei21] I generalized the knot filtration on embedded contact homology, using it to find periodic orbits of surface symplectomorphisms. Part of this work was extended by Bechara Senior-Hryniewicz-Salomaõ [BHS21].
- *Mentorship §2.3:* I mentored two undergraduate summer research projects [DNN$^+$, FHM$^+$22].

Symplectic geometry was revolutionized in the 1980s, when Gromov introduced **pseudoholomorphic** curves in [Gro85] to prove **nonsqueezing**, which showed that symplectic geometry is strikingly different from volume-preserving geometry. A year later, Floer used pseudoholomorphic curves to solve the Arnold conjecture. The invariants Floer constructed inspire today's powerful **Floer homologies** [Flo88], which enabled the proofs of the three-dimensional Weinstein conjecture (Reeb vector fields have periodic orbits) [Tau07], the proof of Property P (nontrivial knots have nontrivial surgeries) [KM04], and the disproof of the Triangulation Conjecture [Man16].

Yet there is a fundamental obstacle in the field: Floer homologies are extraordinarily difficult to compute. Their differentials are constructed using moduli spaces of solutions to nonlinear PDEs, meaning they are defined almost exclusively in analytically generic situations. Meanwhile, computing a Floer differential requires identifying every single PDE solution, which is usually only feasible in the presence of a highly non-generic constraint on the solutions such as a torus action. In short, genericity is the enemy of computability. I work to bridge this gap, creating computational schema for Floer homologies in natural geometric settings with fewer symmetry restrictions and extracting new dynamical information in the settings we currently understand.

Organization: I first provide background, then describe my past, ongoing, and future work in three sections §2-4. Besides §3.3, which requires Definition 2.4, §2-4 may be read independently.

1.1. Background.

A **contact form** is a smooth one-form λ on an odd-dimensional manifold whose kernel hyperplane field is nonintegrable, while a **symplectic form** is a closed smooth two-form ω on an even-dimensional manifold. Both induce volume forms. An essential feature of a contact form is its **Reeb vector field** R, the unique vector field in the kernel of $d\lambda$ with $\lambda(R) = 1$. Its flow preserves λ, and its closed orbits are called **Reeb orbits**.

My work often involves contact and symplectic forms with $d\lambda = \omega$. Exactly what $d\lambda = \omega$ means varies based on the setting (e.g. λ may only be contact when restricted to a submanifold).

Research Program 1.1. When manifolds of adjacent dimensions are topologically compatible and contact/symplectic forms they carry are also compatible, how does this inform their geometry?

My research involves three such types of compatibility: TQFTs and cobordisms, torus and circle actions, and open book decompositions. A $(3+1)$-dimensional topological quantum field theory ("**TQFT**") assigns homology groups to three-manifolds and homomorphisms called **cobordism maps** to 4D cobordisms. I use TQFTs at the chain level to study 4D symplectic cobordisms with contact boundaries, see §2. Circle actions and open books on three-manifolds I explain in §3.

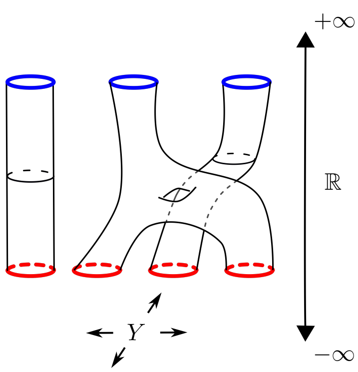


FIGURE 1. A 3D sketch of J-holomorphic curves defining the ECH differential.

One tool I use is **embedded contact homology** ("ECH"), a Floer-type homology theory for a contact three-manifold (Y, λ). It is generated by certain finite sets of Reeb orbits, and its differential is defined by counting **J-holomorphic** (or **pseudoholomorphic**) curves in $\mathbb{R} \times Y$ asymptotic to cylinders over Reeb orbits (Fig. 1). J-holomorphic curves are solutions to a nonlinear Cauchy-Riemann equation on $T(\mathbb{R} \times Y)$, where J is a generalization of $\sqrt{-1}$.

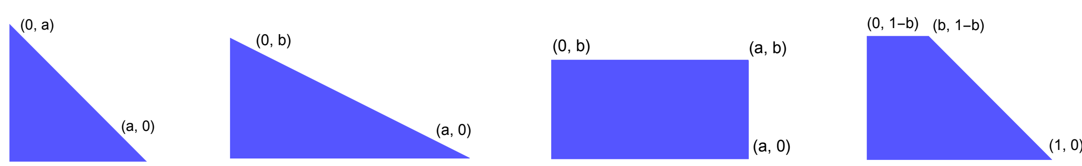
ECH is invariant of λ and depends only on $(Y, \ker \lambda)$ [Tau10]. It is isomorphic to **Seiberg-Witten Floer homology** (HM) and **Heegaard Floer homology** (HF) [KLT, CGH11]. Each has significant strengths when compared to ECH: TQFT cobordism maps are fully defined in HM; HF is more easily computable. Yet the ECH chain complex provides new information about dynamics via the **ECH capacities** (§2) and **knot-filtered ECH** (§4).

Research Program 1.2. Use HM and HF to make the ECH chain complex, ECH capacities, and knot filtration more computable.

2. Symplectic embeddings and ECH capacities

A **symplectic embedding** is an embedding, denoted $X \overset{s}{\hookrightarrow} X'$, which identifies the symplectic forms. Symplectic embeddings are at the heart of a central tenet of symplectic geometry: the rigidity-flexibility dichotomy. A symplectic phenomenon is "rigid" if it is similar to complex geometry, while it is "flexible" if it is constrained only by volume. Rigid and flexible features intertwine throughout the field, and I investigate their interplay via **infinite staircases** (§2.1).

Gromov proved **nonsqueezing** in [Gro85], the groundbreaking result that a $2n$-ball of radius r in $\mathbb{C}^n$ can only embed symplectically into a cylinder $\mathbb{D}^2(R) \times \mathbb{C}^{n-1}$ if $r \leq R$. Nonsqueezing is rigid, as the ball has finite volume while the cylinder's volume is infinite. Conversely, many embedding problems are flexible: [Bir97], [MS12]. Characterizing embeddings is very difficult even for some of the simplest symplectic manifolds [McD09, McD11]. I study embeddings of toric domains, which model 2D physical motion in 4D phase space under a generalized notion of conservation of energy.



(a) *Ball* $B(a)$ (b) *Ellipsoid* $E(a,b)$ (c) *Polydisk* $P(a,b)$ (d) X_b

FIGURE 2. Moment polytopes Ω, labeled by their toric domain X_Ω. All are convex.

Definition 2.1. For Ω a region in the first quadrant of $\mathbb{R}^2$, the **toric domain** (X_Ω, ω_0) is

$$X_\Omega := \{(z_1, z_2) \in \mathbb{C}^2 | (\pi|z_1|^2, \pi|z_2|^2) \in \Omega\}, \quad \omega_0 = dx_1 \wedge dy_1 + dx_2 \wedge dy_2.$$

If Ω is convex, then X_Ω is **convex**. If Ω is a polygon whose sides have rational slopes, X_Ω is **finite type**. We call Ω the **moment polytope** of X_Ω. See Fig. 2 for key examples.

2.1. **Infinite staircases.** Because $X_\Omega \overset{s}{\hookrightarrow} X_{c\Omega'}$ if $c >> 0$, we are interested in the most volume-filling embedding, described by the **ellipsoid embedding function**

$$c_{X_\Omega}(a) := \inf\{c \mid (E(a,1), \omega_0) \overset{s}{\hookrightarrow} (X_{c\Omega}, \omega_0)\}.$$

McDuff and Schlenk in [MS12] found $c_{B(1)}$ has a surprising structure called an **infinite staircase**: $c_{B(1)}$ is either smooth or piecewise linear, with infinitely many nonsmooth points accumulating to a finite limit. Further researchers studied discrete families of c_{X_Ω}: [CFS17, Ush19, CHMP20, Cri]. The unifying **CGHMP conjecture** ([CHMP20, Conj. 1.20]) asserts that (up to scale) there are exactly twelve convex polygons Ω with corners in $\mathbb{Q}^2$ and c_{X_Ω} having an infinite staircase.

Research Program 2.2. With coauthors among Bertozzi, Holm, Magill, Maw, Mwakyoma, McDuff, and Pires, develop machinery computing and classifying c_{X_Ω}; prove the CGHMP conjecture.

We recently completed Research Program 2.2 for the X_bs.

Theorem 2.3 ([MMW22, MPW])**.** *There is a set $B \subset [6,8]$ with:*

(i) $[6,8] \setminus B$ *is homeomorphic to the Cantor set.*
(ii) For each $a_0 \in [6,8] \setminus B$*, there is a unique* b_0 *so that* $c_{X_{b_0}}$ *has an infinite staircase, with steps accumulating to a point with a-coordinate* a_0*.*
(iii) For any other $b \in [0,1)$*,* c_b *is equivalent to one in (ii) or* $c_{1/5}$ *(which has no infinite staircase).*
(iv) The CGHMP conjecture holds for X_b*.*

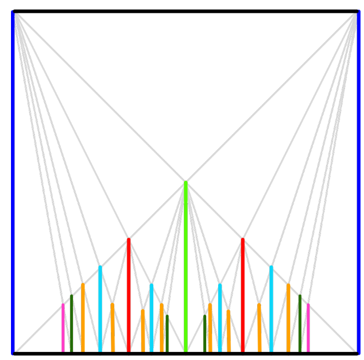


FIGURE 3. A Farey diagram of infinite staircases.

Key infinite staircases generating those in Thm. 2.3 can be represented by the diagonal lines in the Farey diagram (Fig. 3); the complement B of the Cantor set is represented by the vertical lines.

Proving Thm. 2.3 required us to compute huge numbers of ECH capacities.

Definition 2.4 ([Hut11])**.** The **ECH spectrum** of (Y, λ) is

$$0 = c_0(Y,\lambda) < c_1(Y,\lambda) \le c_2(Y,\lambda) \le \cdots \le \infty,$$

where $c_k(Y,\lambda)$ is the length of the shortest homologically essential set of Reeb orbits of ECH grading $2k$. The **ECH capacities** of a symplectic filling (X,ω) of (Y,λ) are $c_k(X,\omega) = c_k(Y,\lambda)$.

ECH capacities obstruct symplectic embeddings:

$$X \overset{s}{\hookrightarrow} X' \iff c_k(X,\omega) \le c_k(X',\omega')\,\forall k. \tag{2.1}$$

The idea of (2.1) is if $\varphi : X \overset{s}{\hookrightarrow} X'$, then $X \setminus \varphi(X')$ is a symplectic cobordism and $c_k(X',\omega') - c_k(X,\omega)$ is at least the area of a J-holomorphic curve in $X \setminus \varphi(X')$ found by the ECH cobordism map [HT13]. By [McD11, Cri19], (2.1) is an equivalence when $X = E(a,b)$ and $X' = X_\Omega$ is convex, implying that each step in c_{X_Ω} arises from a specific J-holomorphic curve.

We computed tens of thousands of ECH capacities to identify the new infinite staircases of Thm. 2.3. Phenomena such as the the fractal structure in Thm. 2.3 (i, ii), the "descending infinite staircases" discovered in [BHM+21], and the symmetries of [MM21] are invisible with fewer.

2.2. **Full CGHMP conjecture.** If X_Ω is convex and finite type then Ω can be obtained from the triangle in Fig. 2 (a) by truncating corners, corresponding to the symplectic blowup of the (closed) symplectic toric manifold $\mathbb{C}P^2$. The X_b require only one blowup, thus are foundational cases for the CGHMP conjecture; our methods also work well for polydisks $P(b,1)$:

Theorem 2.5 ([MPW]). *The CGHMP conjecture holds for polydisks.*

Work in progress by Magill suggests that Thm. 2.3 (i, ii) extends to further blowups by taking a product of Cantor sets.

2.3. **Undergraduate research.** I advised summer research in 2020 (under Jo Nelson) and 2022 (as lead organizer). In 2020, we considered polydisk domains, extending [Hut16a, Thm. 1.5]:

Theorem 2.6 ([DNN$^+$]). *If* $1 \leq a \leq \frac{3}{2}, k \geq 3$, $P(a,1) \overset{s}{\hookrightarrow} E\left(c\frac{2k+1}{2}, c\right) \Leftrightarrow P(a,1) \subset E\left(c\frac{2k+1}{2}, c\right)$.

Our proof used obstructions derived from the genus (rather than the area) of J-holomorphic curve cobordism maps. In 2022, we studied ellipsoid embedding functions with polydisk targets.

Theorem 2.7 ([FHM$^+$22]). *Setting* $\beta = \frac{6+5\sqrt{30}}{12}$, *the function* $c_{P(\beta,1)}$ *has an infinite staircase.*

The significance of Thm. 2.7 is that Usher found countably many $c_{P(\beta,1)}$ with an infinite staircase [Ush19], each with a counterpart staircase c_{X_b} [BHM$^+$21, MM21]. Thm. 2.7 is the first evidence that uncountable fractal structure of Thm. 2.3 (i, ii) respects the polydisk–X_b correspondence.

Research Program 2.8 (With future undergraduates). (i) Extend the "almost toric fibration" methods of Thm. 2.7 to prove [Ush19, Conj. 4.23].
(ii) Optimize the algorithms for genus bounds from [DNN$^+$] to extend Thm. 2.6.

2.4. **Future work: ECH cobordism maps for toric domains.** In [BHM$^+$21, Lem. 92] we related ECH embedding obstructions to those from Seiberg-Witten theory due to McDuff [McD11]. This is useful because ECH capacities are algorithmic (if expensive) to compute, while McDuff's obstructions carry more information yet are not algorithmic and require X_Ω to be finite type.

Our proof of [BHM$^+$21, Lem. 92] is algebraic, but there may be a geometric and more general proof. If $\varphi : E(a,1) \overset{s}{\hookrightarrow} X_\Omega$, the ECH cobordism map ought to count J-holomorphic curves in $X_\Omega \backslash \varphi(E(a,1)$ and recover McDuff's obstructions (which are homology classes of spheres). However, because of extraordinary analytical difficulties, the ECH cobordism map is defined via ECH $\cong$ HM, not by counting curves. The cobordism map does tell us curves exist [HT13], but we only rarely know they are McDuff's spheres [CH18, CHM18].

Question 2.9. When do the ECH curves recover McDuff's curves? Does [BHM$^+$21, Lem. 92] generalize to all convex X_Ω geometrically?

Question 2.9's resolution would have a satisfying application. A crucial result obstructing infinite staircases, [CHMP20, Thm. 1.11], relies on McDuff's methods. The ellipsoids $E(1,b)$ with $b+1/b \in \mathbb{Z}$ (see [CS]) are not finite type, so McDuff's curves do not exist. Using Question 2.9 to translate the proof of [CHMP20, Thm. 1.11] to ECH would rule out infinite staircases for $E(1,b)$, and possibly also for toric domains with nonlinear boundaries. Question 2.9 would also shed more light on the stabilized symplectic embedding conjecture of McDuff [McD18].

2.5. **Future Ph.D. advising.** Boundaries of toric domains generalize to toric lens spaces. My thesis [Wei21] analyzed the ECH chain complexes of toric $L(p,p-1)$s, thus I am interested in:

Research Program 2.10. *(For PhD. students.)* Define "convex" and "concave" toric lens spaces. Find an algorithm to compute their ECH spectra, generalizing [Hut11, Cho16, CCGF$^+$14, Cri19].

Research Program 2.10 would illuminate two phenomena currently only understood for S^3: the geometric meaning of the subleading asymptotics of ECH capacities [CS20] and the utility of ECH capacities to characterize embeddings between symplectic fillings of lens spaces [Lis08, Sta15].

3. The ECH chain complex of S^1-invariant and open book contact forms

In §2 we considered convex regions $X \subset \mathbb{C}^2$ with the symplectic form

$$\omega_0 = dx_1 \wedge dy_1 + dx_2 \wedge dy_2 = d\lambda_0, \quad \lambda_0 = \frac{1}{2}(x_1\, dy_1 - y_1\, dx_1 + x_2\, dy_2 - y_2\, dx_2).$$

Restricted to ∂X (topologically S^3), λ_0 is a contact form. Its Reeb vector field generates a free S^1 action extending to an almost free T^2 action. By far the most is known about such λ (see §2).

Very little is known about the ECH of the many intriguing natural examples of contact manifolds with less symmetry, e.g. the boundary of a Lagrangian product of an ellipse and a disk [Ram17], the Jacobi integral in the circular restricted three-body problem [OR61], or contact structures supported by most open book decompositions [Gir02]. The **Viterbo conjecture** asks whether all symplectic measurements are equal on convex regions in $\mathbb{C}^n$. It is true for regions symmetric under specific circle actions [GRH22, Prop. 1.4]; we would like to generalize this set of actions.

Research Program 3.1. Compute the ECH chain complex and spectrum for non-toric (Y, λ), first systematically studying S^1 actions which do not extend to T^2 actions.

3.1. Prequantization bundles.

These contact manifolds are circle bundles over a surface whose Reeb flow equals the fiber action, and where $d\lambda$ projects to a symplectic form on the base [BW58].

Theorem 3.2 ([NW20]). *Let (Y, λ) be a prequantization bundle. Then $ECH_*(Y, \ker \lambda)$ is the exterior algebra of the homology of the surface.*

Theorem 3.2 first appeared in a $\mathbb{Z}_2$-graded form in the unpublished thesis of Farris [Far11]; Nelson and I completed the J-holomorphic curve analysis and proved the $\mathbb{Z}$-graded version, using HM. Ferreira-Ramos used Thm. 3.2 to compute the ECH capacities of disk bundles [FR21].

An algorithm for the ECH spectrum of general prequantization bundles seems currently out of reach without significant progress on computations of Gromov-Witten invariants. However, there may be some hope when (Y, λ) is symplectically fillable.

Question 3.3. Can we compute the ECH capacities of symplectic fillings of prequantization bundles as was done for symplectic homology in [GS18]?

3.2. Open book decompositions and Seifert fibrations.

An open book decomposition is a fibration over S^1 of a three-manifold minus a link (the **binding**) by Seifert surfaces (the **pages**). A contact form is **adapted** to an open book if its Reeb flow is transverse to the pages and tangent to the binding. An adapted contact form induces a diffeomorphism of the page called the **(Poincaré) return map**, which sends a point to its image under the Reeb flow for the shortest amount of time that point takes to return to the page. An open book is **periodic** or **pseudo-Anosov** if its return map is a periodic or pseudo-Anosov element of the mapping class group of its page.

Many λ adapted to periodic open books have Seifert fibrations generated by the Reeb vector field, a fact Colin–Honda used to prove that their linearized contact homology is nontrivial [CH13]. Nelson and I paired this idea with Kegel-Lange's "orbifold prequantization bundles" [KL21] to prove:

Theorem 3.4 ([NW]). *Let λ be the contact form on S^3 adapted to the open book with binding the $T(2,q), q > 0$ torus knot and periodic return map. With ECC denoting the ECH chain groups,*

$$ECC_*(S^3, \lambda) = ECH_*(S^3, \lambda) = \begin{cases} \mathbb{Z} & \text{if } * = 0, 2, 4, 6, \dots \\ 0 & \text{otherwise.} \end{cases}$$

Pseudo-Anosov open books, such the one on S^3 with the figure-8 as its binding, have much more complicated differentials. Their ECH is work in progress.

3.3. **Future work: HF capacities.** In [CHM$^+$21], Cristofaro-Gardiner–Humiliére–Mak–Seyfaddini–Smith defined the "link spectral invariant" for a Lagrangian link in a surface. Their construction passes through an action filtration on HF which may equal the length filtration on ECH (defining the ECH spectrum, Definition 2.4) by the isomorphism of Colin-Ghiggini-Honda [CGH11].

Question 3.5. Are the link spectral invariants the ECH spectrum? Are they easier to compute?

I am currently generalizing my work on fibered torus knots to compute the ECH capacities of λ adapted to other periodic open books, providing evidence for a positive answer to Question 3.5.

4. Knot-filtered ECH

My 2019 thesis investigated **knot-filtered ECH**, a spectral invariant measuring the linking of ECH generators with a fixed Reeb orbit. It was introduced by Hutchings in [Hut16b] when $H_1(Y) = 0$. If B is a Reeb orbit and $\theta \in \mathbb{R}$ is a parameter depending on $d\lambda$ near B, $ECH^\ell(Y, \ker\lambda, B, \theta)$ is the homology of the subcomplex generated by orbits of linking number at most ℓ with B. I proved:

Theorem 4.1 ([Wei21])**.** *If $b_1(Y) = 0$, $ECH^\ell(Y, \ker\lambda, B, \theta)$ is defined and independent of λ, J.*

I applied Theorem 4.1 to lens spaces $Y = L(p, p-1)$ with B a component of the image of the Hopf link under the quotient $L(p, p-1) = S^3/\mathbb{Z}_p$, computed their knot-filtered ECH, and proved:

Theorem 4.2 ([Wei21])**.** *Contact forms on $L(p, p-1)$ adapted to open books with annulus pages and with unequal irrational binding "rotation numbers" must have a third Reeb orbit γ. The length of γ is bounded from above by a function of its linking numbers with the binding components.*

Bechara Senior–Hryniewicz–Salomão have generalized Thm. 4.2 [BHS21]. In my thesis [Wei21] I used Theorem 4.2 to prove a quantitative smooth version of Franks' theorem [Fra92] that area-preserving homeomorphisms of the annulus have two or infinitely many periodic orbits.

4.1. **Future work.** Understanding the full value of knot-filtered ECH as an invariant will require new computational methods and a significant extension of its foundations.

Research Program 4.3. Prove knot-filtered ECH is defined for all Y and independent of λ, J.

There is a periodic open book on S^3 with the left-handed trefoil as its binding, similar to the one in §3.2. Contact forms adapted to this open book cannot induce Seifert fibrations by [LM04] as they are "overtwisted." We know very little about the ECH of overtwisted contact forms: their chain complexes ECC must be very unusual by [BEVHM12], yet we do not know exactly how.

Question 4.4. What is the knot-filtered ECH of the contact form adapted to the open book with $T(2, \pm 3)$ as its binding? More generally, does knot-filtered ECH detect overtwistedness?

Finally, I hope to relate the ECH knot filtration to the well-studied Heegaard Floer knot filtration.

Question 4.5. Does sending θ to zero in knot-filtered ECH recover knot Heegaard Floer homology HFK, as in the discussion in [CGHH11, §1.2]?

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