# Syllabus for Math 501: Topics in Differential Geometry

Morgan Weiler, Rice University, Spring 2020

**Time:** TTh 10:50-12:05

**Room:** HBH 453

**Office Hours:** HBH 408, M 2-3, T 1-2 and by appointment

**Text:** Lee, *Introduction to Riemannian Manifolds*, Second Edition (Lee RM). WARNING: there is also a first edition! We are not using that. It is also great, but much shorter.

**Prerequisites:** Math 444/539 or equivalent

## Outline

Though we will mainly be using Lee RM, we will also draw from: Lee, *Introduction to Smooth Manifolds* (Lee SM), and Sakai, *Riemannian Geometry*. Other good resources (particularly for the project) are Do Carmo, *Riemannian Geometry*, Petersen, *Riemannian Geometry*, Jöst, *Riemannian Geometry and Geometric Analysis*, and the notes for "Analysis on Manifolds via the Laplacian" by Canzani, taught at Harvard in 2013 and references therein.

We will discuss some necessary topics from the general theory of smooth manifolds. The text has three appendices containing the necessary background material, but if you are not comfortable with the basics on manifolds you may want to also do some reading outside of class. I can suggest references, but I will first suggest Lee SM ;)

I also suggest you read Chapter 1 of Lee RM on your own in the first few weeks of class.

The tentative outline is as follows:

- General manifolds: integral curves and flows (Lee SM Ch. 9), Lie derivatives (Lee SM Ch. 9, 12, 14), distributions and foliations (Lee SM Ch. 19), contact distributions (Lee SM Ch. 22), Lie groups (Lee SM Ch. 7, 8, 20, 21). We will not be covering the indicated chapters in their entirety, just the sections relevant to our class.
- Riemannian basics: Riemannian metrics (Lee RM Ch. 2), model Riemannian manifolds (Lee RM Ch. 3), connections, covariant derivatives, geodesics, parallel transport (all Lee RM Ch. 4), Levi-Civita connection (Lee RM Ch. 5), exponential map of a Riemannian manifold (Lee RM Ch. 5), geodesics and distance (Lee RM Ch. 6), curvature Lee RM Ch. 7).
- As time permits: Riemannian submanifolds, eigenfunctions of the Laplacian and curvature, the Laplace spectrum and length spectrum, Weyl's law.

## Assessment

Your grade will be computed according to the following formula:

$$55\%(\text{hw}) + 5\%(\text{leading hw discussion}) + 35\%(\text{talk}) + 5\%(\text{attendance at talks})$$

**Homework:** I will assign approximately 5 problems weekly. They will be due Thursdays at the beginning of class, starting 1/23. They will be graded out of one point, with credit being awarded for solid work towards a solution. You are encouraged to work together and to come to office hours to discuss homework problems with me.

You will also lead a discussion in the first 10 minutes of class on at least one Thursday during the semester. The topic will be a homework problem of your choice (though I reserve the right to rule problems out). You do not have to know how to solve the whole problem, but you should have a good understanding of the issues involved and be able to moderate a discussion amongst your fellow students. If you want, you can structure this as a 5 minute presentation with 5 minutes for questions, but you do not have to.

**Project:** In the last few weeks of class, you will give a 25 minute talk on a topic of your choice. Guidelines will appear later in the semester. You'll first clear your topic with me.

Potential topics for your project (off the top of my head and not exhaustive): minimal surfaces, the Gauss-Bonnet theorem, Jacobi fields, comparison theory, major theorems on the relationship between curvature and topology (see Lee RM Ch. 12), the sphere theorem, connections and curvature on principal bundles, index theorems, Minkowski/Finsler/Lorentz metrics, relationships with contact and symplectic geometry, the very basics of Ricci flow...

You could also delve deeply into the proof of a theorem or even include historical information about the development of a theory.