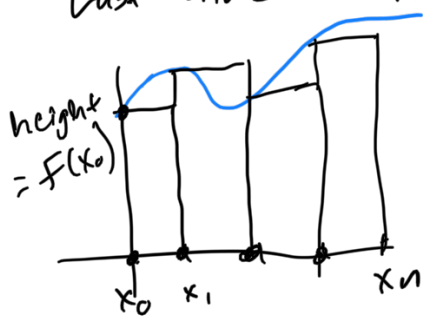
Lec 2/10

First HW assigned tom. afternoon

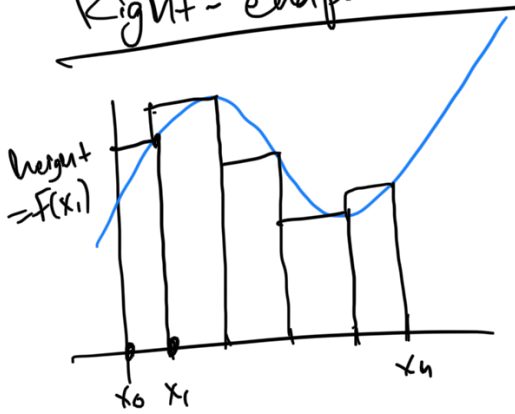
Last time: approximating areas w/ left-endpoint approxima



$$\rightarrow \sum_{i=1}^{n} f(x_{i-1})\overbrace{(x_i - x_{i-1})}^{\text{length of } [x_{i-1}, x_i]}$$

left-endpoint $[x_{i-1}, x_i]$

Right-endpoint approximation

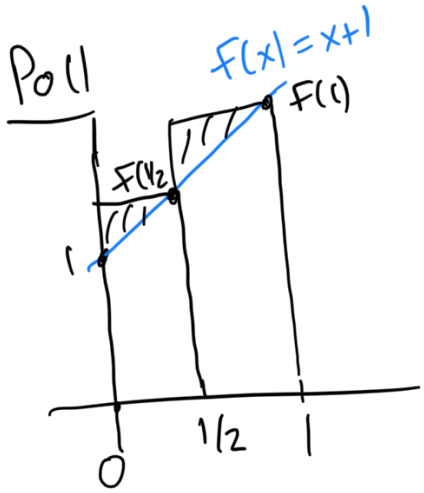


$$\sum_{i=1}^{n} f(x_i)\,(x_i - x_{i-1})$$

area between 0, 1, under f

$n=2$, right-endpoint, using same length intervals

Poll



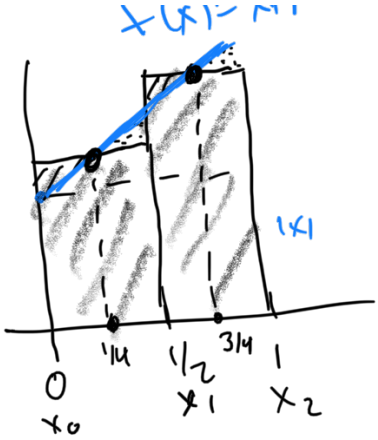
$$f(1/2)\,(1/2 - 0) + f(1)\,(1 - 1/2)$$

$$(1/2 + 1)(1/2) + (1+1)(1/2)$$

$$\frac{3}{4} + 1 = \frac{7}{4}$$

Midpoint rectangle approximation

$f(x) = x+1$ on $[0, 1]$



$n=2$, ...

$$\sum_{i=1}^{n} f\left(\frac{x_{i-1}+x_i}{2}\right)(x_i - x_{i-1})$$

midpoint of $[x_{i-1}, x_i]$

$$f\left(\frac{x_0+x_1}{2}\right)(x_1-x_0) + f\left(\frac{x_1+x_2}{2}\right)(x_2-x$$

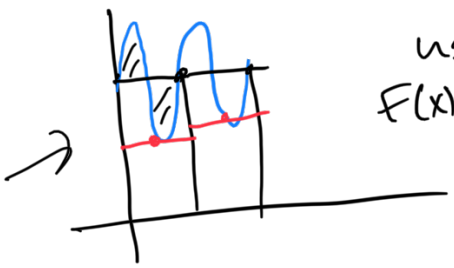
$$f\left(\frac{0+1/2}{2}\right)(1/2-0) + f\left(\frac{1/2+1}{2}\right)(1-1/2)$$

$$= f(1/4)(1/2) + f(3/4)(1/2)$$

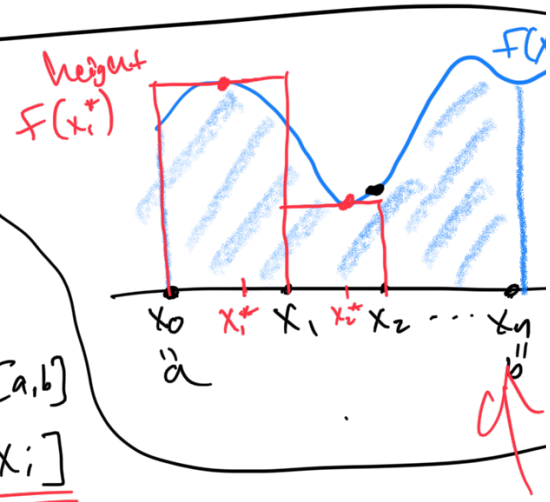
use $f(x)=x+1$

$$= \frac{5}{4}(1/2) + 7/4\,(1/2)$$

$$\frac{5}{8} + \frac{7}{8} = \frac{12}{8} = \frac{3}{2}$$



Riemann Sums



Def f fn on $[a,b]$

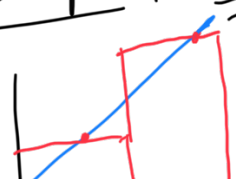
$\{x_0, x_1, \dots, x_n\}$ partition of $[a,b]$

x_i^* is any pt in $[x_{i-1}, x_i]$

then
$$\sum_{i=1}^{n} f(x_i^*)(x_i - x_{i-1})$$

is a Riemann sum for f over $[a,b]$

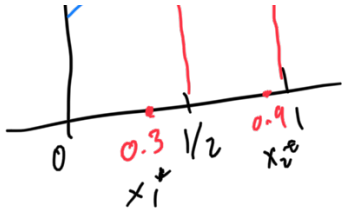
Example: $f(x) = x+1$ on $[0,1]$, $n=2$



Riemann sum

$$f(x_1^*)(x_1 - x_0) + f(x_2^*)(x_2 - x_1)$$

$$= \ldots (1/2)$$



$$= f(0.3)(1/2) + f(0.9)(\ldots$$
$$= 1.3\,(1/2) + (1.9)(1/2)$$
$$= 0.65 + 0.95 = 1.6$$

- If $x_i^* = x_{i-1}$ for all i, then the Riemann sum agrees with left-endpoint approximation.
- If $x_i^* = x_i$ $\longrightarrow$ right-endpoint approx
- If $x_i^* = \dfrac{x_{i-1} + x_i}{2}$ $\longrightarrow$ midpoint approx

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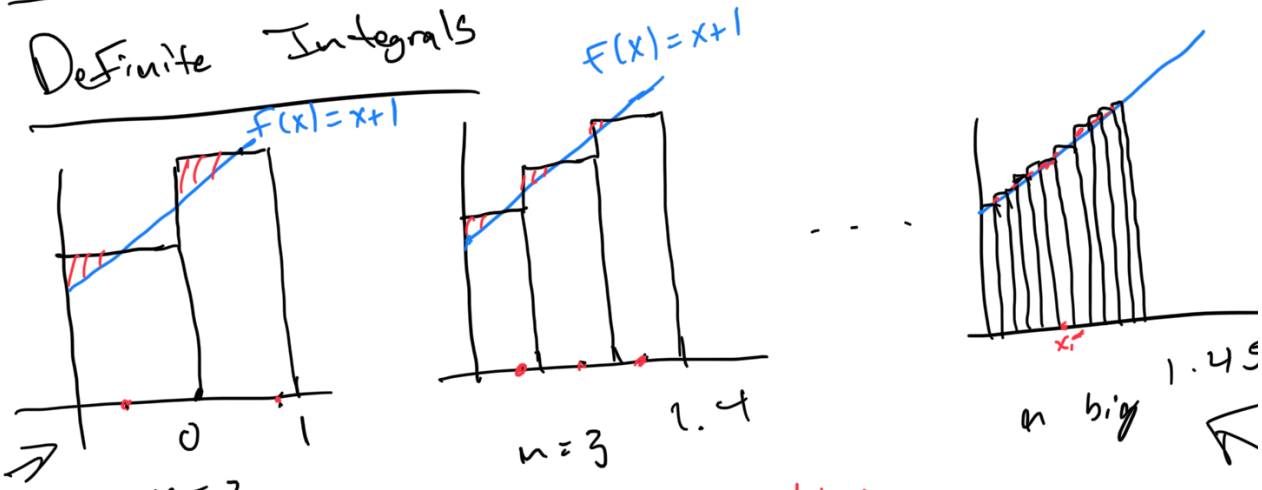
Definite Integrals



Error in approximation to area gets smaller as $n \to \infty$

Def: f function on $[a,b]$

The <u>definite integral</u> of f from a to b is

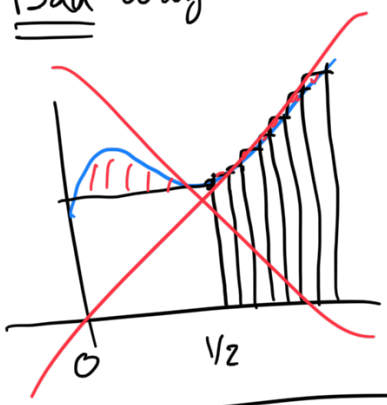
$$\int_a^b f(x)\,dx = \lim \sum^{n} f(x_i^*)\,\underbrace{(x_i - x_{i-1})}_{\Delta x_i}$$

$\rightarrow \int_a \; f(x)\,dx \qquad n \to \infty \qquad i=1$ ↑ could be neg/pos positive

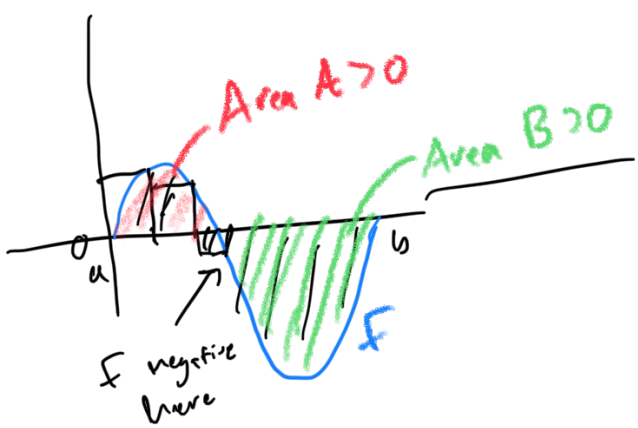
where $a = x_0, x_1, \ldots, x_n = b$ is a regular (all intervals have same length) partition of $[a,b]$,

$\rightarrow x_i^*$ is in $[x_{i-1}, x_i]$

Bad way to approx. area



Geometric interpretation of definite integral: "Net signed area"



$$\int_a^b f(x)\,dx = A - B$$