Last time: definite integral

Relationship between derivatives and integrals

Q: What happens when you take derivative of an integral?

\[ \int_0^x f(t)\, dt \]

\[ \text{Area under } F \text{ between } 0 \text{ and } x! \]

Think of as a function of \( x \)

\[ \Delta \text{Area} \approx \Delta x \cdot f(x) \]

\[ \frac{\Delta \text{Area}}{\Delta x} \approx f(x) \]

Derivative of area

Fundamental Theorem of Calculus, \( F \) \( \in \) \([a,b]\)

\[ \frac{d}{dx} \int_a^x f(t)\, dt = f(x) . \]

\[ \text{differentiation undoes 'integration'} \]

Ex: \( f(t) = t^2 \)
\[
\frac{d}{dx} \left( \int_1^x t^2 \, dt \right) = f(x) = x^2
\]

Example 2: \[
\frac{d}{dx} \int_1^x t^2 \, dt
\]

\[
F(x) = \int_1^x t^2 \, dt
\]

Fund. Thm., Part 1: \[F'(x) = \frac{d}{dx} F(x) = x^2\]

Want: \[\frac{d}{dx} F(x^2) = 2x \cdot F'(x^2) = 2x \cdot x^2 = 2x^5\]

Poll: \[
\frac{d}{dx} \int_1^x t^3 \, dt
\]

\[
F(x) = \int_1^x t \, dt
\]

\[F'(x) = \frac{d}{dx} F(x) = x\]

Want: \[\frac{d}{dx} F(x^2) = 2x \cdot F'(x^2) = 2x \cdot x^2 = 3x^5\]

Power rule for derivs

**Fundamental Theorem of Calculus, Part 2**

\[F\text{ function on an interval } [a,b]\]

\[F'\text{ exists and is continuous, then} \]

\[f(b) - f(a) = \int_a^b f(t) \, dt\]
\[ -2 \int_a^b F'(t) \, dt = F(b) - F(a) \]

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integration undoes differentiation
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**Interpretation:**
- \( F \) distance of a car from 0
- \( F' \) velocity of car
- distance traveled between time \( a \) to \( b \) = \( F(b) - F(a) \)

\[
= \int_a^b F'(t) \, dt
\]

Very helpful for computing integrals

**Ex:** \( \int_0^2 t \, dt \)

Want: \( F \) s.t. \( F'(t) = t^1 \)

Recall power rule: \( \frac{d}{dt} t^n = n \cdot t^{n-1} \) \((n \neq 0)\)

\[
\frac{d}{dt} t^2 = 2t
\]

\[
\frac{d}{dt} \frac{1}{2} t^2 = t
\]

"\( \frac{1}{2} t^2 \) is the antiderivative of \( t \)"

\( F(t) = \frac{1}{2} t^2, \ F'(t) = t \)

\[
\int_0^2 t \, dt = \int_0^2 F'(t) \, dt = F(2) - F(0)
\]
Final Thm, part 2

\[ = \frac{1}{2} (2)^2 - \frac{1}{2} (0)^2 \]
\[ = \frac{1}{2} \times 4 = 2 \]

Poll

Evaluate \( \int_{-1}^{1} t^2 \, dt \)

Want: \( F(t) \) such that \( F'(t) = t^2 \)

\[ \frac{d}{dt} t^3 = 3 t^2 \quad \text{Finding anti-derivative} \]

\[ \int_{-1}^{1} t^2 \, dt = \frac{1}{3} t^3 \bigg|_{-1}^{1} \quad \text{Fundamental Theorem} \]

\[ = \frac{1}{3} (1^3 - (-1)^3) \]
\[ = \frac{1}{3} + \frac{1}{3} = \frac{2}{3} \]

Anti-derivatives: If \( n \neq -1 \)

Power Rule: \( x^n \) \( \frac{1}{n+1} x^{n+1} + C \)

\( F(t) = t^2 \)
\[ \int x^2 \, dx = \frac{x^3}{3} + C \]