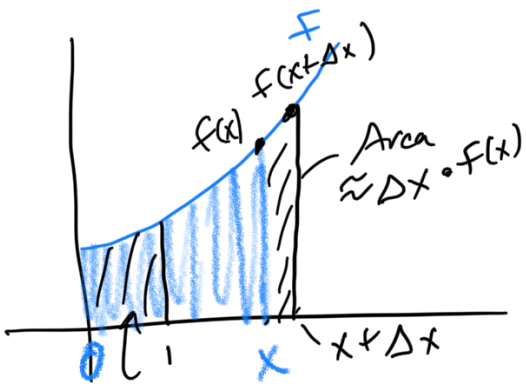
# Lec 2/15

## Last time: definite integral

Relationship between derivatives + integrals

Q: What happens when you take derivative of an integral?




Area under F between 0 and x!

$$\int_0^x F(t)\,dt$$

think of as a function of x

Area $= f(1)$

$$\Delta \text{Area} \approx \Delta x \cdot F(x)$$

divide

derivative of area $= \dfrac{\Delta \text{Area}}{\Delta x} \approx F(x)$

**Fundamental Theorem of Calculus (Part 1): F continuous fn on [a,b]**

$$\frac{d}{dx}\int_a^x F(t)\,dt = F(x).$$

"differentiation undoes integration"

Ex: ① $f(t) = t^2$

$$\frac{d}{dx}\int_1^x t^2\,dt = f(x) = x^2$$

Ex (2) $\frac{d}{dx}\int_1^{x^2} t^2\,dt$

$$F(x) = \int_1^x t^2\,dt$$

Fund Thm, part 1 : $F'(x) = \frac{d}{dx}F(x) = x^2$

Want: $\frac{d}{dx}F(x^2)$

Chain Rule for derivs $\rightarrow$ $= 2x\,F'(x^2) = 2x(x^2)^2 = 2x^5$

Poll : $\frac{d}{dx}\int_1^{x^3} t\,dt$

$$F(x) = \int_1^x t\,dt$$

$F'(x) = \frac{d}{dx}F(x) = x$

Want: $\frac{d}{dx}F(x^3) = \left(\frac{d}{dx}x^3\right)\cdot F'(x^3)$ Chain Rule

$= 3x^2 \cdot x^3 = 3x^5$ Power rule for derivs

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**Fundamental Theorem of Calculus, part 2**

$F$ function on an interval $[a,b]$

$F'$ exists and is continuous, then

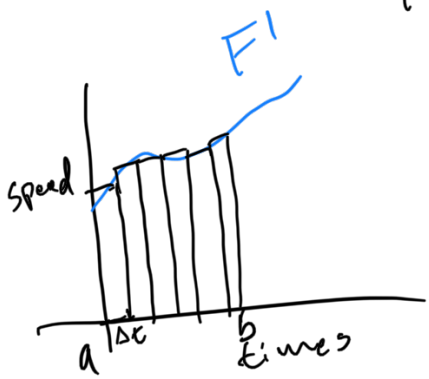
$F(b) - F(a)$

$$\int_a^b F'(t)\,dt = F(b) - F(a)$$

"integration undoes differentiation"

<u>Interpretation</u>: $F$ distance of a car from $0$

$F'$ velocity of car




distance traveled between time $a, b$ $= \underline{\underline{F(b) - F(a)}}$

$$= \int_a^b F'(t)\,dt$$

<u>Very helpful for computing integrals</u>

<u>Ex</u>: Compute $\int_0^2 \underline{\underline{t}}\,dt$


Want: $\underline{F}$ s.t. $F'(t) = t^1$

Recall power rule: $\frac{d}{dt} t^n = n t^{n-1}$ $(n \neq 0)$

$$\frac{d}{dt} t^2 = 2t$$

$$\frac{d}{dt} \frac{1}{2} t^2 = t$$

"$\frac{1}{2} t^2$ is the <u>antiderivative</u> of $t$"

$F(t) = \frac{1}{2} t^2$, $\quad F'(t) = t$

$$\int^2 t\,dt = \int_0^2 F'(t)\,dt = \underline{F(2) - F(0)}$$

Fund Thm, part 2

$$= \frac{1}{2}(2)^2 - \frac{1}{2}(0)^2$$
$$= \frac{1}{2}\,4 = 2$$

## Poll

Evaluate $\int_{-1}^{1} t^2\,dt$

Want: $F(t)$ such that $F'(t) = t^2$

$$\frac{d}{dt} t^3 = \underline{\underline{3}}\,t^2 \quad \Big\} \text{ Finding anti-derivative}$$

$$\longrightarrow \frac{d}{dt} \underline{\underline{\frac{1}{3}}}\,t^3 = t^2$$

new notation

$$\int_{-1}^{1} t^2\,dt \underset{\text{Fund Thm}}{=} \frac{1}{3}t^3 \Big|_{-1}^{1}$$

upper limit

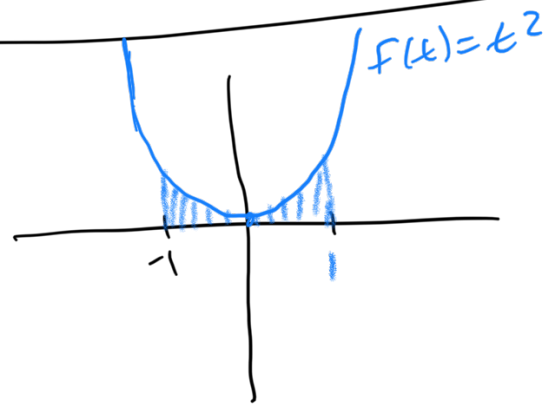
lower limit

$$= \frac{1}{3}(1)^3 - \frac{1}{3}(-1)^3$$
$$= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Anti-derivative Power Rule: If $n \neq -1$

$x^n$

antideriv: $\frac{1}{n+1}x^{n+1} + C$ ← constant



$$\int x^2 dx = \frac{x^3}{3} + C$$