**Lecture 3/15: Volumes of solids of revolution**

Ex: What is the volume of the solid of revolution formed by revolving \( y = 2x \) on \([0, 1]\) about the \(x\)-axis?

\[
\int_0^1 \pi (2x)^2 \, dx = 4\pi \int_0^1 x^2 \, dx = 4\pi \left. \frac{x^3}{3} \right|_0^1 = \frac{4\pi}{3}
\]

**Disk method**: Volume of solid of revolution formed by revolving \( y = f(x) \) over \([a, b]\) about the \(x\)-axis is:

\[
\int_a^b \pi (f(x))^2 \, dx.
\]

Ex: Revolve \( y = x^2 \) over \([0, 2]\) about the \(x\)-axis:

\[
\text{Volume} = \int_0^2 \pi (x^2)^2 \, dx
\]
PoI1: Find volume of solid formed by revolving \( y = \sqrt{x} \) over \([1, 4]\) about \(x\)-axis.

**Disk method:**

\[
\pi \int_{1}^{4} (\sqrt{x})^2 \, dx = \pi \int_{1}^{4} x \, dx = \frac{\pi}{2} x^2 \bigg|_{1}^{4} = \frac{15\pi}{2}
\]

Ex1: Find volume of solid formed by revolving \( x = \sqrt{4-y} \) over \([0, 4]\) about \(y\)-axis.

**Disk method:**

Volume \(= \int_{0}^{4} \pi (\sqrt{4-y})^2 \, dy \)

\[
= \pi \int_{0}^{4} (4-y) \, dy = \left[4\pi y - \frac{\pi y^2}{2}\right]_{0}^{4} = 15\pi
\]
Solids formed by revolving region between two curves.

Ex

What is volume of solid formed by revolving region between $y = \sqrt{x}$ and $y = 1$ over $[1, 4]$ about the x-axis.

Volume = $\int_{1}^{4} \pi ((\sqrt{x})^2 - 1^2) \, dx$

$= \pi \int_{1}^{4} (x - 1) \, dx = \pi \left[ \frac{x^2}{2} - x \right]_{1}^{4}$

$= \pi (8 - 4) - \pi \left( \frac{1}{2} - 1 \right)$

$= 4 \pi + \frac{\pi}{2} = \frac{9\pi}{2}$
Washer Method

If \( f(x), g(x) \) functions with \( f(x) \geq g(x) \) for \( x \) in \([a, b]\), then volume of solid formed by revolving region between \( f(x), g(x) \) over \([a, b]\) about \( x\)-axis is

\[
\int_a^b \pi \left( (f(x))^2 - (g(x))^2 \right) \, dx.
\]

(3) Find volume of solid formed by revolving region bounded by \( \sqrt{x} \) and \( \frac{1}{x} \) over interval \([1, 3]\) about \( x\)-axis.

Intersection points:

Solve \( \sqrt{x} = \frac{1}{x} \)

\( x^{1/2} = \frac{1}{x} \)

\( x^{3/2} = 1 \)

So \( \sqrt{x} \geq \frac{1}{x} \) for \( x \) in \([1, 2]\)

Volume

\[
= \int_1^3 \pi \left( (\sqrt{x})^2 - \left( \frac{1}{x} \right)^2 \right) \, dx
\]

\[
= \pi \left[ \frac{3}{2} \left( x - \frac{1}{x^2} \right) \right]_1^3
\]

\[
= \pi \left( \frac{x^2}{2} + x^{-1} \right)\bigg|_1^3
\]

\[
= -\pi \left( \frac{1}{a} \pm 1 \right) - \pi \left( 1 \pm 1 \right)
\]
Ex: Find volume of solid formed by revolving region bounded by $y = 4-x$ and the x-axis about the line $y = -2$ over $[0, 4]$

Volume = \[ \pi \int_0^4 (6-x)^2 - 2^2 \, dx \]