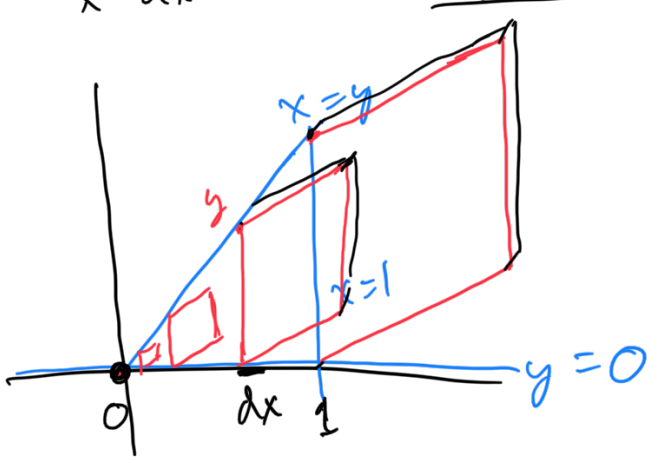
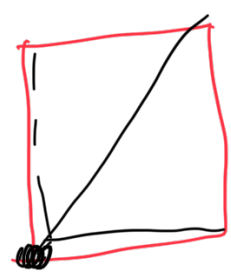
Lec 3/24

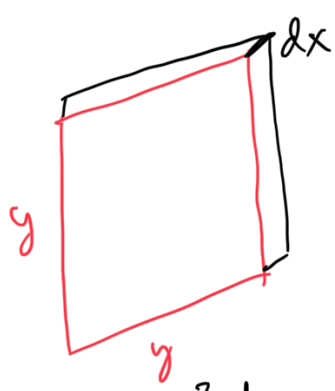
Last time: solids of known cross-section

Poll Calculate volume of solid whose base is the region bounded by $x=1$, $y=0$, $x=y$, and whose cross sections perpendicular to the x-axis are squares.



Rotated over:





$\text{vol} = y^2\,dx$

$= x^2\,dx$

(since $y = x$ is

Volume of solid

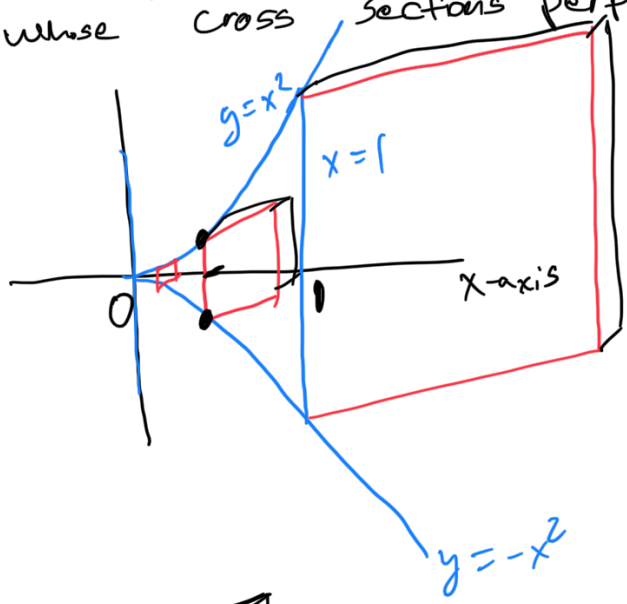
$$= \int_0^1 x^2\,dx$$

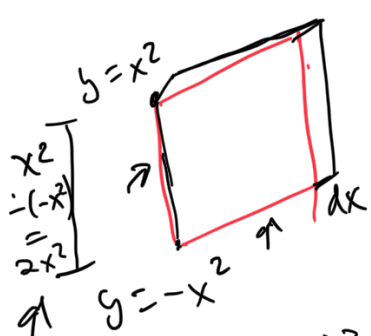
$$= \frac{x^3}{3}\Big|_0^1$$

$$= \frac{1}{3} - 0 = \frac{1}{3}$$

the top way)

Ex Calculate volume of solid whose base is the region bounded above by $y = x^2$, below by $y = -x^2$, and on sides by $x = 0$, $x = 1$, and whose cross sections perpendicular to x-axis are squares





$$\text{Vol} = (2x^2)^2\,dx$$

Volume of solid

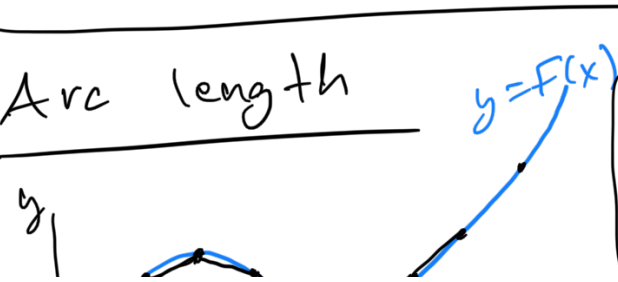
$$= \int_0^1 (2x^2)^2\,dx$$

$$= 4\int_0^1 x^4\,dx$$

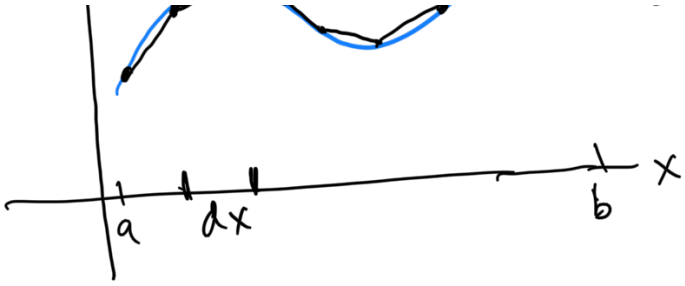
$$= 4\,\frac{x^5}{5}\Big|_0^1$$

$$= \frac{4}{5}$$

Arc length

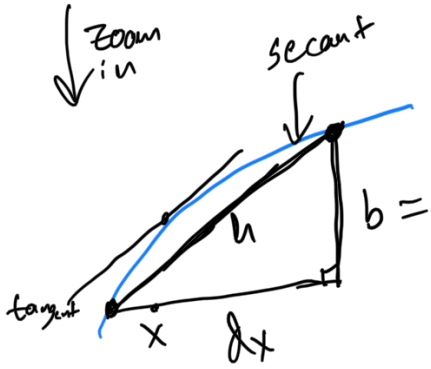


Q: What is length of the curve under $y = f(x)$ between a and b?



Idea: Approximate a curve by many small line segments.

Zoom in



$b = f'(x)\,dx$

Pythagorean theorem:

$$\text{length line segment} = h = \sqrt{(dx)^2 + (f'(x)dx)^2}$$

$$= dx\sqrt{1+(f'(x))^2}$$

Slope of secant line $\approx$ slope of tangent line $= f'(x)$

$$\text{slope} = \frac{b}{dx}$$

$$f'(x) = \frac{b}{dx}$$

$$\Rightarrow b = f'(x)dx$$

Arc length: sum of lengths of the line segments (as dx gets small

$$= \int_a^b \sqrt{1+(f'(x))^2}\,dx$$

EX What is arc length of the curve $y = 2x^{3/2}$ over $[0,1]$?

$$f(x) = 2x^{3/2}, \quad f'(x) = 2\left(\tfrac{3}{2}\right)x^{1/2} = 3x^{1/2}$$

Power rule for derivatives

$$\text{Arc length} = \int_0^1 \sqrt{1+(f'(x))^2}\,dx$$

$$= \int_0^1 \sqrt{1+(3x^{1/2})^2}\,dx = \int_0^1 \sqrt{1+9x}\,dx$$

$$\rightarrow \frac{1}{9}\int \sqrt{1+9x}\; 9\,dx$$

u-sub:

$u = 1+9x$

$du = 9\,dx$

$$= \frac{1}{9}\int \sqrt{u}\; du$$

$$= \frac{1}{9}\int u^{1/2}\, du$$

$$= \frac{1}{9}\,\frac{2}{3}\,u^{3/2}$$

$$= \frac{2}{27}\left(1+9x\right)^{3/2}$$

$$= \frac{2}{27}\left(1+9x\right)^{3/2}\Big|$$

$$= \frac{2}{27}(10)^{3/2} - \frac{2}{2}$$

Poll Find arc length of the curve $y = 3x+1$ over interval $[0,1]$.

$f(x) = 3x+1$, $f'(x) = 3$

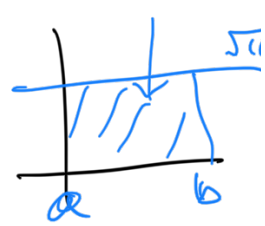
$$\text{Arc length} = \int_0^1 \sqrt{1+(f'(x))^2}\; dx$$

$$= \int_0^1 \sqrt{1+3^2}\; dx$$

$$= \int_0^1 \sqrt{10}\; dx$$

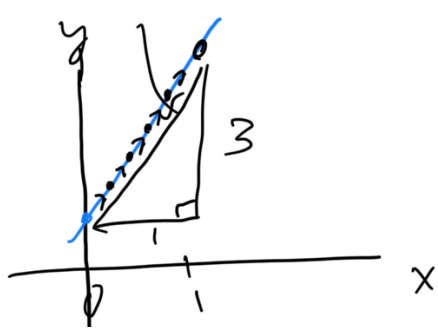
$$= \sqrt{10}\; x \Big|_0^1 = \sqrt{10}$$

area $= \sqrt{10}\,(b-a)$



$\sqrt{10} = \sqrt{10}\, x^0$

$\sqrt{3^2+1^2} = \sqrt{10}$



<u>Ex</u> Compute arc length of $y = x^2$ over the interval $[0,1]$. ← Can't do yet!

$f(x) = x^2, \quad f'(x) = 2x$

$$\text{Arc length} = \int_0^1 \sqrt{1+(f'(x))^2}\,dx$$

$$= \int_0^1 \sqrt{1+(2x)^2}\,dx$$

$$= \int_0^1 \sqrt{1+4x^2}\,dx$$

Try u-sub

$u = 1+4x^2$

$du = 8x\,dx$

Don't know how to do yet!