Lec 3/24

Last time: solids of known cross-section

Poll Calculate volume of solid whose base is the region bounded by \( x = 1, \ y = 0, \ x = y, \) and whose cross sections perpendicular to the \( x \)-axis are squares.

Rotated over:

Volume of solid
\[
\int_0^1 x^2 \, dx
\]

\[
= \frac{x^3}{3} \bigg|_0^1
\]

\[
= \frac{1}{3} - 0 = \frac{1}{3}
\]
EX Calculate volume of solid whose base is the region bounded above by \( y = x^2 \), below by \( y = -x^2 \), and on sides by \( x = 0 \), \( x = 1 \); and whose cross sections perpendicular to the \( x \)-axis are squares.

Volume of solid:

\[
\int_0^1 (2x^2)^2 \, dx
\]

\[
= 4 \int_0^1 x^4 \, dx
\]

\[
= 4 \left[ \frac{x^5}{5} \right]_0^1
\]

\[
= \frac{4}{5}
\]

Q: What is length of the curve under \( y = f(x) \) between \( a \) and \( b \)?
Idea: Approximate a curve by many small line segments.

Pythagorean theorem:

\[ b = f'(x) \, dx \]

Length of line segment:

\[ h = \sqrt{(dx)^2 + (f'(x) \, dx)^2} \]

\[ = dx \sqrt{1 + (f'(x))^2} \]

Arc length: sum of lengths of the line segments as \( dx \) gets small

\[ = \int_a^b \sqrt{1 + (f'(x))^2} \, dx \]

Example: What is arc length of the curve \( y = 2x^{3/2} \) over \([0, 1]\)?

\( f(x) = 2x^{3/2} \), \( f'(x) = 2(\frac{3}{2})x^{1/2} = 3x^{1/2} \)

Power rule for derivatives:

\[ \frac{d}{dx} (2x^{3/2}) = 3x^{1/2} \]

Arc length:

\[ = \int_0^1 \sqrt{1 + (3x^{1/2})^2} \, dx \]

\[ = \int_0^1 \sqrt{1 + 9x} \, dx \]
\[
\int_0^1 \sqrt{1+9x} \, dx = \frac{1}{9} \int \sqrt{u} \, du = \frac{1}{9} \int u^{1/2} \, du = \frac{1}{9} \left[ \frac{2}{3} u^{3/2} \right] = \frac{2}{27} (1+9x)^{3/2}
\]

**Problem** Find arc length of the curve \( y = 3x + 1 \) over interval \([0,1] \).

\( f(x) = 3x + 1 \), \( f'(x) = 3 \)

Arc length = \( \int_0^1 \sqrt{1 + (f'(x))^2} \, dx \)

\[
= \int_0^1 \sqrt{1 + 3^2} \, dx = \int_0^1 \sqrt{10} \, dx
\]

\( \sqrt{3^2 + 1^2} = \sqrt{10} \)

\( \sqrt{10} \times 1_0^1 = \sqrt{10} \)

**Area**

\[\text{Area} = \frac{1}{2} (6 - a)\]

\( a = \sqrt{10} \times x \)
Ex Compute arc length of \( y = x^2 \) over the interval \([0, 1]\).

\[ f(x) = x^2, \quad f'(x) = 2x \]

\[
\text{Arc length} = \int_0^1 \sqrt{1 + (f'(x))^2} \, dx
\]

\[
= \int_0^1 \sqrt{1 + (2x)^2} \, dx
\]

Don't know how to do yet!