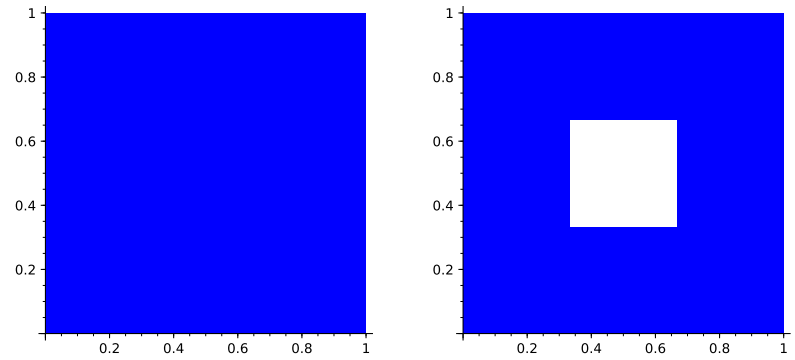
Homework 7: MAT 331

Due: 11:59pm, Wednesday, 4/15/2020

Part of this homework is written and part is programming. You will submit everything via Blackboard. You will upload one `pdf` file called `prob1prob2.pdf` that contains written solutions to both Problems 1 and 2 below. You can make this `pdf` file in any number of ways, including using a word processor and exporting to `pdf`, or by (legibly) scanning/photographing hand-written solutions.

For Problem 3, you will submit a single Jupyter notebook file called `prob3.ipynb`.
Grader for this assignment: Zhuang Tao, `zhuang.tao@math.stonybrook.edu`

Background: All the problems concern a fractal called the *Sierpinski carpet* $\mathcal{C}$, defined as the limit of the sets $\mathcal{C}_n$, which are shown below in blue. We start with $\mathcal{C}_0$ a solid square, and then obtain $\mathcal{C}_i$ by dividing $\mathcal{C}_{i-1}$ into squares, and removing the middle (1/9)th of each square. The limit object $\mathcal{C}$ consists of all points of the original square that are never removed.



(a) $\mathcal{C}_0$ (b) $\mathcal{C}_1$

(c) $\mathcal{C}_2$ (d) $\mathcal{C}_3$

Figure 1: Iterates converging to $\mathcal{C}$.

Problems

1. Compute the area of the Sierpinski carpet $\mathcal{C}$ by computing the limit of the areas of the $\mathcal{C}_i$.

2. Compute the fractal dimension of the Sierpinski carpet $\mathcal{C}$ (see HW6 for the definition of fractal dimension.)

3. (Worth 10 points, double the usual) Write a program to draw iterates of the Sierpinski carpet. You should make a function

   `sierp_carp(basept, size, iter)`

   that takes in `basept` of the form (x, y) and outputs a copy of $\mathcal{C}_{\texttt{iter}}$ of size proportional to `size` and whose left endpoint is at (x, y).

   Submit a single Jupyter notebook file called `prob3.ipynb` containing your program.