Lec 1: Mandlebrot Set

Represents information about all Julia sets of polynomials $f(z) = z^2 + c$, for all c complex number.

$$M = \{c : \lim_{n \to \infty} |f_c^{\circ n}(0)| \neq \infty\}$$ ← Subset of $\mathbb{C}$ cpx plane

$$= \{c : 0 \text{ is } \underline{\text{not}} \text{ in escaping set for } f_c\}$$

Alternate characterization

$$M = \{c : J(f_c) \text{ is connected}\}$$

e.g. $0 \in M$ since $J(f_0) = J(z^2)$ = unit circle, which is connected

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Fixed/periodic points

Fixed pt of f: z s.t. $f(z) = z$

Periodic pt of f: z s.t. $f^{\circ n}(z) = z$ for some n

smallest n is called period

(Note: a fixed pt is a periodic point with period 1)

<u>Examples</u>: $f(z) = z^2$

$f(0) = 0 \Rightarrow 0$ is a fixed pt.

Fixed pt eqn: $f(z) = z$

$$z^2 = z$$

solve: $z^2 - z = z(z-1) = 0$

$\implies z = 0, \; z = 1$

Periodic points of order 2: $f^{\circ 2}(z) = z$ (and aren't fixed pts)

$$f^{\circ 2}(z) = f(z^2) = (z^2)^2 = z^4$$

Need to solve: $z^4 = z$

$$0 = z^4 - z = z(z^3 - 1)$$

$\implies z = 0, \; z^3 = 1$

$z^3 = 1 \to$ 3 solns