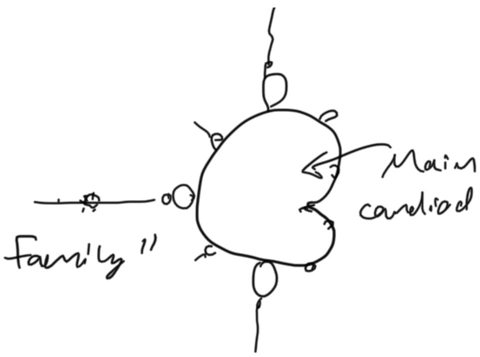
Lec 2: Mandlebrot set

Represents information about all Julia sets of form $F_c(z) = z^2 + c$ "quadratic family" for $c \in \mathbb{C}$ cpx plane.



$$M = \{ c : 0 \text{ is not in escaping set for } F_c(z) = z^2 + c \}$$

$$= \{ c : \lim_{n \to \infty} |F_c^{\circ n}(0)| \neq \infty \}$$

Alternate characterization: $F_c(z) = z^2 + c$

$$M = \{ c : J(F_c) \text{ is } \underline{\text{connected}} \}$$

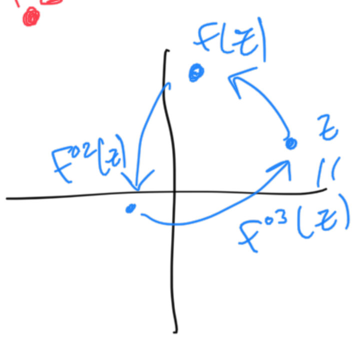
M is "connectedness locus"

Fixed pts and periodic pts

Fixed pt for F: z s.t. $F(z) = z$

Periodic pt for F: z s.t. $F^{\circ n}(z) = z$ for some n.

Minimal such n is <u>order</u> of periodic pt.

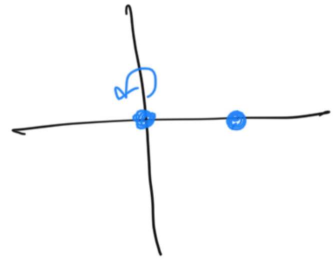


. z periodic of order 3

Useful to study: easier understand long term behavior under iteration

How to find fixed/periodic pts:

E.g. $f(z) = z^2$



Fixed pt equation:

$$f(z) = z$$

$$z^2 = z$$

Solve: $z^2 - z = 0$

$$z(z-1) = 0$$

$$\Rightarrow z = 0, \quad z = 1$$

Periodic pts: of order 2

$$f^{\circ 2}(z) = z \leftarrow$$

$$(z^2)^2 = z \leftarrow$$

$$z^4 = z$$

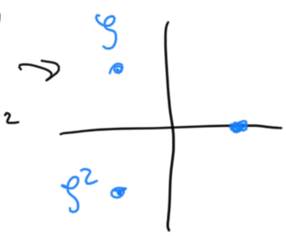
Solve: $z^4 - z = 0$ (degree 4)

Factor: $z(z^3 - 1) = 0$

Solutions: $z = 0$, $z^3 = 1$ $\Rightarrow$

$\downarrow$

$z = 1, \zeta$, or ζ^2



Order 2 pts: ζ, ζ^2