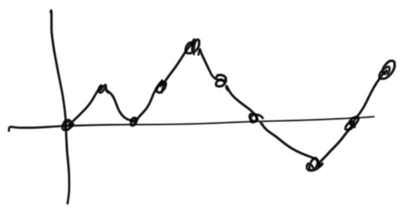
Last time: Brownian motion as limit of random walk: H T H T T H

↑ ↓



Random variable taking values $-1, 1$ each w/ probability $1/2$.

Take many of these: $X_1, X_2, \ldots$

(Vert) Position of random walk (at time n):

$X_1 + X_2 + \ldots + X_n$

Random variables:

Examples: 1 $\{0, 1\}$ random uniform variable (randint(0,1))

2 $\{-1, 1\}$ uniform
$2 \cdot$ randint$(0,1) - 1$

3 $\{0, 1\}$ w/ prob of $0 = 1/3$
and prob of $1 = 2/3$
(non-uniform)

discre

4 $[0, 1]$ uniform random Continu
every real num between 0 and 1 is equally likely

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uniform

X_1 {0, 1} random

- $X_1 + X_1$ new random variable
  {0, 2} uniform

- X_1, X_2 be two {0,1} uniform r.v.

  $X_1 + X_2$ don't know what this is
  unless we know how X_1, X_2
  are related

  Independent: $X_1 + X_2$
  0,1 0,1

	X_1	X_2	$X_1 + X_2$
equally likely	0	0	0
	1	0	1
	0	1	1
	1	1	2

$X_1 + X_2$ a
{0, 1, 2}
0 w/ prob 1/4
1 w/ prob 1/2
2 w/ prob 1/4
r.v.

For random walk, want $X_1, X_2, \ldots, X_n$
{-1, 1} uniform r.v. w/ X_i and X_j
if $i \neq j$.

Vert pos: $\sum_{i=1}^{n} X_i$

Mean (Expected Value) of X_i is

$$\frac{1}{2}(-1) + \frac{1}{2}(1) = 0$$

Study: $\sum_{i=1}^{n} X_i$ and also

averages $\frac{1}{n} \sum_{i=1}^{n} X_i$

Law of Large Numbers:

$X_1, X_2, X_3, \ldots$ any random variables from the same distribution, and independent (and $Ave[X_i]$ is finite)

Then $\frac{1}{n}\sum_{i=1}^{n} X_i \to Ave(X_i)$

doesn't depend on i, since all X_i have same distribution

as $n \to \infty$.