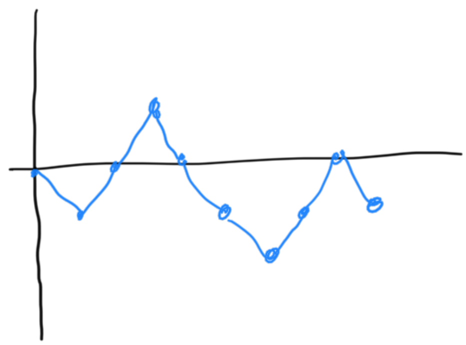
Lecture 2:

Last time: Brownian motion as limit of random walk

$$-1, 1, 1, -1, -1, -1, 1, 1, -1$$



vertical position is a sum of a bunch of -1's and 1's.

Examples of random variables:

① $\{0, 1\}$ uniform random variable

0 w/ prob 1/2

1 w/ prob 1/2

Generated by randint(0,1)

$\text{Ave} = \frac{1}{2}(0) + \frac{1}{2}(1) = \frac{1}{2}$

② $\{-1, 1\}$ uniform r.v.

-1 w/ prob 1/2

1 w/ prob 1/2.

2 · randint(0,1) − 1

$\text{Ave} = \frac{1}{2}(-1) + \frac{1}{2}(1) = 0$

③ $\{0, 1\}$ non-uniform

0 w/ prob 1/3

1 w/ prob 2/3

$\text{Ave} = \frac{1}{3}(0) + \frac{2}{3}(1) = \frac{2}{3}$

discrete r.v.

④ $[0, 1]$ uniform r.v. prob landing in $[a, b]$ $(a > 0, b < 1)$

(all x w/ $0 \le x \le 1$

is $b-a$

$Ave = \frac{1}{2}$ random() — continuous r.v.

Sums of r.v.

- Let X_1 be a $\{0,1\}$ unif. r.v.

  $X_1 + X_1$ is $\{0, 2\}$ unif. r.v.

- Let X_1, X_2 be two $\{0,1\}$ unif r.v.

  What is $X_1 + X_2$? Need to know how X_1, X_2 are related/correlated?

Independent:

X_1	X_2	X_1+X_2
0	0	0
1	0	1
0	1	1
1	1	2

Independence $\Rightarrow$ all 4 equally likely

$X_1 + X_2$ is $\{0, 1, 2\}$

0 w/ prob $1/4$

1 w/ prob $1/2$

2 w/ prob $1/4$

r.v. (non-uniform)

$\frac{1}{2}(X_1 + X_2)$ is $\{0, 1/2, 1\}$

0 w/ prob $1/4$

$1/2$ w/ prob $1/2$

1 w/ prob $1/4$

Q: How does average

$$\frac{1}{n}(X_1 + X_2 + \dots + X_n)$$ behave

where X_i are independent r.v.

?

from <u>same</u> distribution:

Expect: this tends to

$\text{Ave}(X_i)$

(aka Expected value)

<u>Law of Large Numbers</u>

Let $X_1, X_2, \ldots$ <u>independent</u> r.v.

w/ <u>same</u> distribution (s.t. $\text{Ave}(X_i)$ is finite)

Then $\frac{1}{n}(X_1 + \cdots + X_n) \to \text{Ave}(X_i)$

as $n \to \infty$.

doesn't depend on i, since all X_i have same distribution

(Demonstrated for X_i $\{0,1\}$ uniform)