Lecture 1: X_i $\underline{\text{iid}}$ r.v.

Q: How much does $\underline{(X_1 + \dots + X_n)}$ deviate
n

from $Ave(X_i)$ in terms of n?

Think of $Y_n = \frac{X_1 + \dots + X_n}{n}$ ⇐ sample means

as random variables.

$$Ave(Y_n) = Ave\left(\tfrac{1}{n}(X_1 + \dots + X_n)\right)$$
$$= \tfrac{1}{n}\left(Ave(X_1) + \dots + Ave(X_n)\right)$$
$$= \tfrac{1}{n}\left[Ave(X_i) \cdot n\right] = Ave(X_i)$$

How to measure closeness of Y_i to its average?

$$Var(Y_i) = \underline{Ave\left(\left(Y_i - Ave(Y_i)\right)^2\right)}$$

Bigger the more Y_i varies

(standard deviation is square-root of variation)

<u>Variance of sample means</u>

$$Var(Y_i) = Ave\left(\left(Y_i - Ave(Y_i)\right)^2\right)$$
$$= Ave\left(\left(\tfrac{1}{n}(X_1 + \dots + X_n) - Ave(X_i)\right)^2\right)$$
$$= Ave\left(\left(\frac{(X_1 - Ave(X_1)) + \dots + (X_n - Ave(X_n))}{n}\right)^2\right)$$
$$= Ave\left((X_1 - Ave(X_1))^2 + \dots + (X_n - Ave(X_n))^2\right.$$

$$\ldots \left(\frac{\ldots}{n^2} \right.$$

Cross terms are all 0, since X_i, X_j indep $i \neq j$

$$\left. + \frac{(X_1 - Ave(X_1))(X_2 - Ave(X_2)) + \ldots}{n^2} \right)$$

$$= Ave\left(\frac{Var(X_1) + \ldots + Var(X_n)}{n^2} + 0 \right)$$

$$= \frac{1}{n^2}\, n\, Var(X_i) = \frac{1}{n} Var(X_i)$$

$$\rightarrow \boxed{Var\left(\frac{X_1 + \ldots + X_n}{\sqrt{n}} \right) = Var(X_i)}$$

Central Limit Thm

$X_1, X_2, \ldots$ indep, identically distributed r.v.

(plus some condition on existence of $Ave(X_i)$, $Var(X_i)$)

Assume $Ave(X_i) = 0$, $Var(X_i) = 1$

Then $\frac{X_1 + \ldots + X_n}{\sqrt{n}} \rightarrow$ Standard Normal distribution (Bell curve) as $n \rightarrow \infty$

(convergence in distribution)