

Lecture 2: Poisson Distribution

- ① Big batch of cookie dough w/ chocolate chips. How many chocolate chips in one cookie?
- ② Stand by highway. Count how many cars go by in one minute.
(times that cars come by are independent, happens w/ some given average frequency)
- ③ Meteor strikes on the moon. How many new craters does moon develop every million years?

Common characteristics:

- Answer is non-neg integer
- Some average frequency/density/intensity λ (lambda)

All these examples accurately modeled by Poisson dist.

We'll define Poisson as limit of Binomial distributions

Binomial (n, p) :
positive integer n probability in $(0, 1)$

Have a coin that has prob p of landing heads. Flip n times, record number of heads. Flips are independent.

Eg. Binomial $(1, 1/2)$ is a single fair coin flip.
constant $(0, 1)$

Binomial $(10, 1/2)$ 10 fair coin flips
of heads
sum of 10 copies of $\text{randint}(0, 1)$.

Cookie dough analogy: Divide a fistful
of dough into n equal sized pieces,
each of which has either 0 or 1 chips.
If one fistful has, on average, λ chips,
then think of each of the n pieces as
having 1 chip w/ prob λ/n , and
0 chips otherwise

Chips in one chunk: modeled by coin
flip w/ prob λ/n

Chips in cookie: sum of n independent
instances of above
Binomial $(n, \lambda/n)$

Poisson (λ) is limit of Binomial $(n, \lambda/n)$ as $n \rightarrow \infty$