Homework 02: MAT 364

Collaboration Policy: You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly. It is generally easy to convert a file from some other format, such as .docx, to .pdf.

1. Which closed intervals $[a, b] \subset \mathbb{R}^1$ have the property that $[a, b] \cap \mathbb{Q}$ is *open* relative to $\mathbb{Q}$. Here $\mathbb{Q}$ is the set of rational numbers?

2. Let $A \subset \mathbb{R}^2$ be defined by $A = D^2((0, 0), 1) \cup D^2((5, 5), 1)$. Find a subset $B \subset A$ that is open relative to A, closed relative to A, and $B \neq \emptyset, A$.

3. (*Kinsey 2.25*) Prove that the composition of two continuous functions is continuous.

4. Show, directly from Definition 2.15, that the function $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} 0 \text{ if } x \text{ is rational} \\ 1 \text{ if } x \text{ is irrational} \end{cases}$$

   is not continuous.

5. Find an example of a continuous function $f : D \to R$ and an open set U in D such that $f(U)$ is *not* open in R.

6. Find an example of a continuous function $f : D \to R$ and a closed set U in D such that $f(U)$ is *not* closed in R. Find such an example in which furthermore f is surjective, i.e. $f(D) = R$.

(*Optional Challenge Problem*) Find an example of a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that for any $\epsilon > 0$ the interval (x, ϵ) contains points y, z with $f(y) < f(x)$ and $f(z) > f(x)$.