Homework 5: MAT 364

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly. It is generally easy to convert a file from some other format, such as .docx, to .pdf.

1. Let X be a topological space for which the topology is discrete (i.e. every subset is open). Prove, for any topological space Y and any function $f : X \to Y$, that f is continuous.

2. Kinsey 3.13 ("Flea and comb" space is connected)

3. Prove that if a subspace A of $\mathbb{R}^n$ is compact in the sense of Kinsey Definition 3.17, then it is also compact in the sense of Kinsey Definition 2.23 (she calls this "sequential compactness", but be aware that this term is typically defined somewhat differently).

4. (Kinsey 3.21) Let X be a compact topological space and $f : X \to Y$ that is surjective (i.e. $f(X) = Y$). Prove that Y is compact.

5. Consider the Zariski topology from HW04, in the case of $\mathbb{R}$. Show that this topology is not Hausdorff.

   (Hint: you can describe the topology fairly explicitly in this one-dimensional case - a similar topology on a different set is discussed in Kinsey Ch. 3.3.)

6. (Kinsey 3.26) Let X be a Hausdorff space, and Y a compact subset of X. Prove that Y is closed (in X).

Optional Problem: Can one put a topology on $\mathbb{R}$ such that the resulting topological space is homeomorphic to $\mathbb{Q}$ (with the subspace topology it inherits as a subset of $\mathbb{R}$)?