Homework 6: MAT 364

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly. It is generally easy to convert a file from some other format, such as .docx, to .pdf.

1. (Kinsey 3.29) Show that the product of two T_i spaces is a T_i space, for $i = 0, 1, 2$.

2. Let X_i, $i = 1, 2$, be spaces, and $\mathcal{B}_i$ a basis for X_i. Prove that

$$\mathcal{B} := \{B_1 \times B_2 : B_1 \in \mathcal{B}_1, B_2 \in \mathcal{B}_2\}$$

   is a basis for the product topology on $X_1 \times X_2$.

3. Let X_i, $i = 1, 2$, be spaces, and let A_i be a subset of X_i. There are two natural ways to define a topology on $A_1 \times A_2$: (i) give A_i the subspace topology it inherits from the space X_i, and then give $A_1 \times A_2$ the product topology, or (ii) give $X_1 \times X_2$ the product topology, and then give $A_1 \times A_2$ the subspace topology it inherits from the space $X_1 \times X_2$. Show that these two topologies coincide.

4. Given a space X with an equivalence relation $\sim$, we defined the quotient topology on $X/\sim$ by saying that a subset U is open iff $p^{-1}(U)$ is open in X, where $p : X \to X/\sim$ is the natural quotient map. Show that this does in fact define a topology, i.e. the collection of such U satisfies the three axioms of a topology.

5. (Kinsey 3.35) Show that if X is connected and $\sim$ is an equivalence relation on X, then $X/\sim$ is connected.

6. Find an example of a Hausdorff space X and an equivalence relation $\sim$ on it, such that $X/\sim$ is not Hausdorff.

Optional problem: Is every space X homeomorphic to the quotient of some $\mathbb{R}^n$ by some equivalence relation $\sim$?