Homework 7 : MAT 364

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly. It is generally easy to convert a file from some other format, such as .docx, to .pdf.

1. Prove that following planar diagram describes a topological space homeomorphic to a torus

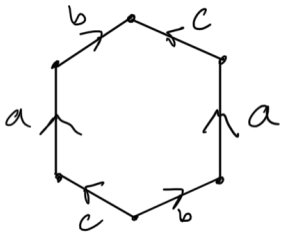

Figure 1:

2. Consider a Möbius strip cut along the dashed blue lines as shown below. What is the resulting surface?

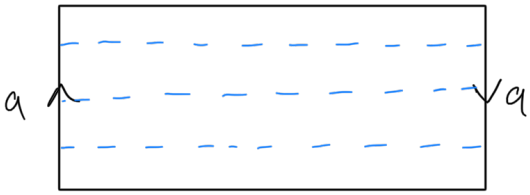

Figure 2:

3. Find a planar diagram (i.e. a collection of polygons with certain pairs of edges identified) that represents a space homeomorphic to a two-holed torus (this is more formally known as a *genus* 2 *surface*).

4. Kinsey Exercise 4.4

5. Prove (using the definition given in Lec for 10/12) that a (finite) CW-complex is a compact topological space.

6. Do an experiment with a paper Möbius strip (ala the YouTube video), and describe the result.

Optional challenge problem: Find a way to embed a two-holed torus into $\mathbb{R}^3$ so that it has order 3-symmetry, i.e. there exists a rotation of $\mathbb{R}^3$ through 120 degrees about some axis that preserves the two-holed torus (takes each point on the surface to some point on the surface).