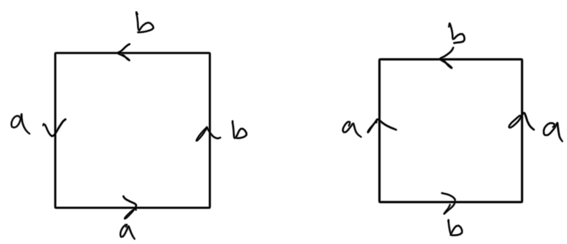
Homework 8: MAT 364

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly. It is generally easy to convert a file from some other format, such as .docx, to .pdf.

1. Is an open subset of an n-manifold always an n-manifold? How about a closed subset?

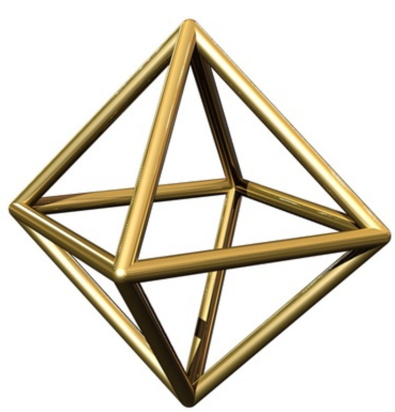
2. Show that the two planar diagrams below represent homeomorphic surfaces.



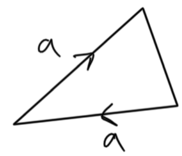
3. Suppose that X, Y are compact, connected surfaces that can be represented by planar diagrams. Prove that the connect sum $X\#Y$ is orientable iff both X, Y are orientable.

4. Find a planar diagram that represents $\mathbb{P}^2\#T$, the connect sum of a projective plane and a torus.

5. Consider the surface embedded in $\mathbb{R}^3$ depicted below. Since it is a connected, compact surface, the theorem on classification of surfaces implies that it is homeomorphic to a sphere, connect sum of n tori, or connect sum of n projective planes. Which of these is it (and if not the sphere, what is the n)?



6. Show that the planar diagram below represents a surface homeomorphic to a Möbius strip.



Optional task: Make a triangle out of paper (or some other material) and glue two sides together as in the last problem above to form a Möbius strip. Hint: Using a different triangle may make the gluing easier to achieve.