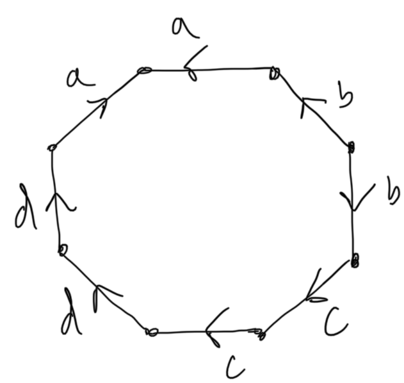
Homework 9: MAT 364

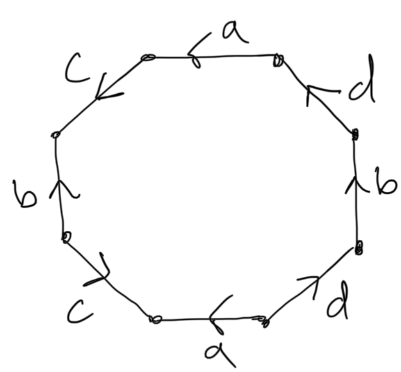
Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly. It is generally easy to convert a file from some other format, such as .docx, to .pdf.

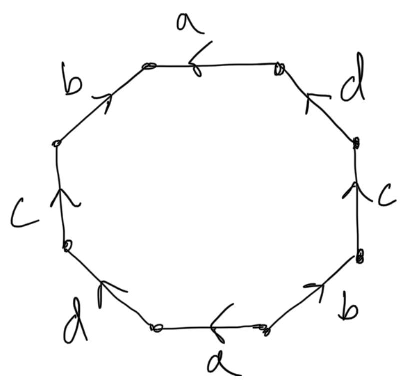
1. The following diagram represents a connected, compact surface, so the theorem on classification of surfaces implies that it is homeomorphic to a sphere, connect sum of n tori, or connect sum of n projective planes. Which of these is it (and if not the sphere, what is the n)?



2. The following diagram represents a connected, compact surface, so the theorem on classification of surfaces implies that it is homeomorphic to a sphere, connect sum of n tori, or connect sum of n projective planes. Which of these is it (and if not the sphere, what is the n)?



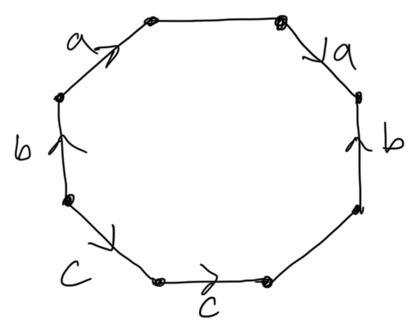
3. The following diagram represents a connected, compact surface, so the theorem on classification of surfaces implies that it is homeomorphic to a sphere, connect sum of n tori, or connect sum of n projective planes. Which of these is it (and if not the sphere, what is the n)?



4. Prove that any compact connected surface is homeomorphic to a connect sum of n tori and k projective planes, where $k \leq 2$. (We allow $n = 0$ and/or $k = 0$; when both are zero, we have an the "empty" connect sum, which is by convention we take to be the sphere).

5. (Kinsey 4.26) The cylinder and the Möbius strip are connected, compact surfaces with boundary. Describe each as a sphere, a connect sum of n tori, or a connect sum of n projective planes, with a finite number of discs removed.

6. The following diagrams represents a connected, compact surface with boundary. Describe it as a sphere, a connect sum of n tori, or a connect sum of n projective planes, with a finite number of discs removed.



Optional Challenge Problem: Consider the *configuration space of unordered pairs of distinct points on a circle* S^1 given by the quotient space:

$$\big((S^1 \times S^1) - \{(x, x) : x \in S^1\}\big) \,/\, \sim,$$

where $(x_1, y_1) \sim (x_2, y_2)$ iff $(x_1, y_1) = (x_2, y_2)$ or $(x_1, y_1) = (y_1, x_1)$. This space is homeomorphic to surface (possibly with boundary). Determine which one it is.