Homework 10: MAT 364

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Submission: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly. It is generally easy to convert a file from some other format, such as .docx, to .pdf.

1. Fix positive integers k and f. Consider a polyhedron with f faces such that all faces are k-gons. How many edges does this polyhedron have (in terms of k, f)?

2. Find a polyhedral representation of the torus by gluing together some number of cubes, and compute $v - e + f$ for this polyhedron.

3. Find a CW-complex that represents the n-dimensional closed ball $\overline{D^n}$. For this CW-complex, compute $e_0 - e_1 + e_2 - e_3 \pm \cdots$, where e_i is the number of i-cells (also called i-facets). Does your answer depend on n?

4. We define *real projective n-space* $\mathbb{P}^n$ to be the quotient space of the closed ball $\overline{D^n(0,1)}$ by the equivalence relation $\sim$, where $x \sim y$ iff $x = y$, or $x = -y$ and $\|x\| = 1$ (i.e. we are quotienting by the "antipodal map" on the boundary sphere). Note that this generalizes the definition of the (real) projective plane $\mathbb{P}^2$. Find a CW-complex that represents real projective n-space. For this CW-complex, compute $e_0 - e_1 + e_2 - e_3 \pm \cdots$, where e_i is the number of i-cells (also called i-facets).

5. Kinsey Exercise 5.2 (you only need to find the trees with 6 or fewer vertices)

6. Consider graphs G with n vertices that have no self-loops (i.e. an edge with the same two vertices as endpoints), and no multiple-edges (more than one edge with the same pair of vertices as endpoints). What is the minimum possible value of the Euler characteristic $\chi(G)$ among such graphs G?

Optional Challenge Problem: Consider a two-holed torus (connect sum of two tori) embedded in $\mathbb{R}^3$ in the usual way. Find two disjoint closed simple (i.e. non self-intersecting) loops on the surface that are linked together in $\mathbb{R}^3$. Linked means that if we keep just the two loops and remove the rest of the surface, there is no way to continuously deform the two loops so that they are far away from each other (without having them pass through one another).