# Homework 11 : MAT 364

**Collaboration Policy** : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

**Submission**: Upload a `.pdf` file using the page for this assignment in Blackboard. You may produce this either (i) electronically, or (ii) by hand, legibly, and then scanned, legibly. It is generally easy to convert a file from some other format, such as .docx, to .pdf.

1. Prove, using planar diagrams, that if $S_1, S_2$ are compact surfaces (without boundary), then

$$\chi(S_1 \# S_2) = \chi(S_1) + \chi(S_2) - 2.$$

2. Let $X$ be a CW-complex. A *subcomplex* $A$ of $X$ is a subset of $X$ that is a union of open cells of $X$ such that the closure of each cell in $A$ is contained in $A$. This means that for each cell $B_i$ in $A$, the attaching map $f_i$ for this cell has image in $A$. Such an $A$ naturally inherits a CW-complex structure. Show that if $A_1, A_2$ are subcomplexes of $X$ such that $A_1 \cap A_2$ is also a subcomplex of $X$, then

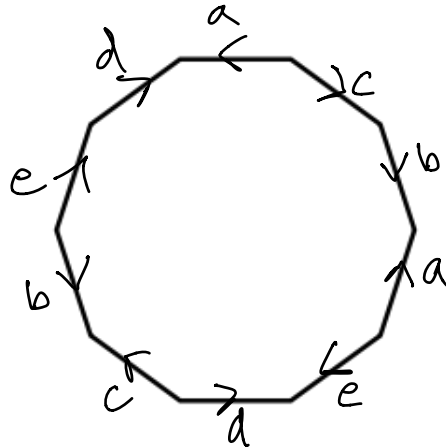$$\chi(A_1 \cup A_2) = \chi(A_1) + \chi(A_2) - \chi(A_1 \cap A_2).$$

3. The following diagrams represents a connected, compact surface. Describe it as a sphere, a connect sum of $n$ tori, or a connect sum of $m$ projective planes.
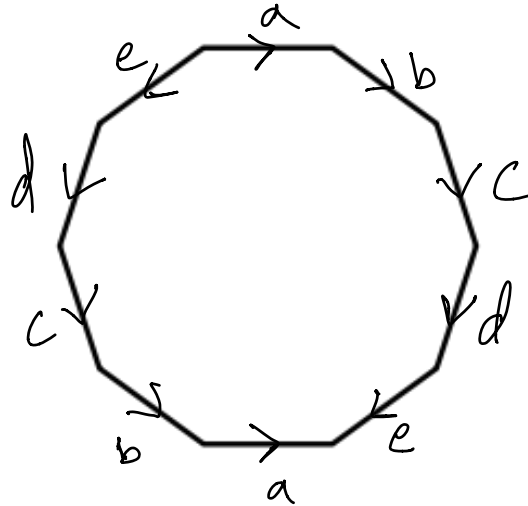


4. Let $X$ be the surface obtained from a planar diagram that is a $2k$-gon, with each pair of opposite sides identified in an opposing manner (i.e. one arrow goes clockwise, the other

counter-clockwise). Describe it as a sphere, a connect sum of $n$ tori, or a connect sum of $m$ projective planes. The planar diagram above is the case $k = 5$.

5. The following diagrams represents a connected, compact surface. Describe it as a sphere, a connect sum of $n$ tori, or a connect sum of $m$ projective planes.

6. The following diagrams represents a connected, compact surface with boundary. Describe it as a sphere, a connect sum of $n$ tori, or a connect sum of $m$ projective planes, with a finite number of discs removed.