Point-set Topology in $\mathbb{R}^n$

→ $\mathbb{R}^n = \{(x_1, .., x_n) : x_i \text{ is a real number}\}$

$\mathbb{R}^1$ is the real line

$\mathbb{R}^2$ is xy-plane

Notion of distance

$x = (x_1, \dots, x_n)$, $y = (y_1, \dots, y_n)$

dist from x to origin $\|x\| = \sqrt{x_1^2 + \dots + x_n^2}$

$d(x,y) = \|x-y\| = \sqrt{(x_1-y_1)^2 + \dots + (x_n-y_n)^2}$

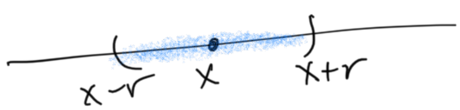
(Open) disc/ball, $x \in \mathbb{R}^n$, $r > 0$ real num.

(x: center)

$$D^n(x,r) := \{y \in \mathbb{R}^n : d(x,y) < r\}$$

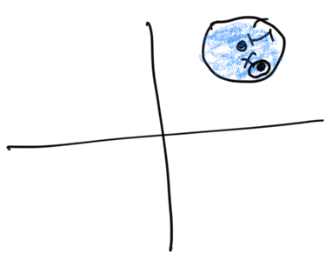
aka (open) disc neighborhoods

$n=1$



$D(x,r) = (x-r, x+r)$

$n=2$

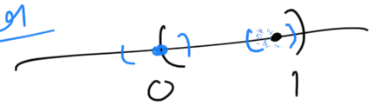


Def: Let $A \subset \mathbb{R}^n$, we say $x \in A$ is an <u>interior point</u> of A if there is some disc $D = D(x,r)$ thats contained in A ie.

$B \subset A$

Ex: ① $A = (0,1) \subset \mathbb{R}$

open



What are interior pts?

All pts in $(0,1)$

② $A = (0,1] = \{y : 0 < y \le 1\}$

not open: 1 is in A, 1 is not an interior pt

Interior pts $= (0,1)$

Def: Say $A \subset \mathbb{R}^n$ is <u>open</u> if every pt in A is an interior pt of A.