Limit Points, Closed Sets, Limits of Sequences

Def $A \subset \mathbb{R}^n$, say $x \in \mathbb{R}^n$ is a limit pt of A
if every disc centered at x $(B(x,r))$
intersects A,
ie $B(x,r) \cap A \neq \emptyset$ — empty set.

Ex. every elt $x \in A$ is a limit pt of A
(since $x \in B(x,r)$, $x \in A$
$x \in B(x,r) \cap A$)

Ex $A = D^2((1,1), 1/2)$

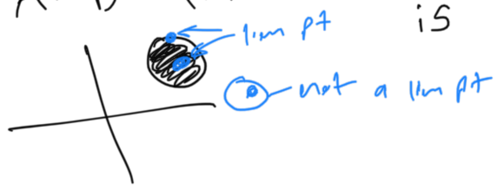
$S((1,1), 1/2)$

limit pts of A $= \{x \in \mathbb{R}^2 : d((1,1),x) \leq 1/2\}$

Def: $A \subset \mathbb{R}^n$, say A is <u>closed</u> if
A contains all its limit points.

Ex: $A = D^2((1,1), 1/2)$ (open disc) is <u>not</u> closed,
because pts in $S((1,1), 1/2)$ are
lim pts of A, but are not in A

Ex: $A = \overline{D}^2(x, R) := \{y \in \mathbb{R}^2 : d(x,y) \leq r\}$
is <u>closed</u>.



Ex. $A = (0, 1]$

$= \{x : 0 < x \leq 1\}$

not open (see last time)

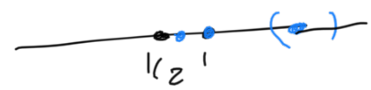
not closed: 0 is a limit pt of A not in A

Limits of sequences $\quad x_1, x_2, x_3, \ldots \quad \{x_n\}$

Def If sequence $x_1, x_2, \ldots \in \mathbb{R}^n$, then a limit pt of the sequence is any point $y \in \mathbb{R}^n$ such that for any $r > 0$ $D^n(y, r)$ contains x_i for infinitely many i.

Ex $x_1 = 1,\ x_2 = 1/2,\ x_3 = 1,\ x_4 = 1/2, \ldots$

What are limit pts in $\mathbb{R}$?



- 1 is a limit pt of seq (since any $D^1(1, r)$ contains x_i for all odd i)
- $1/2$ is limit pt (any $D^1(1/2, r)$ contains x_i for all even i