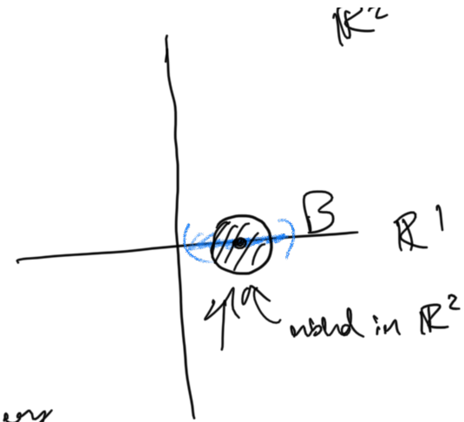
Relative Neighborhoods

$(0,1) \subset \mathbb{R}^1$ is open in $\mathbb{R}^1$



Q: Is $(0,1)$ open as subset of $\mathbb{R}^2$?

A: No. No pt of $(0,1)$ is an interior pt, since every disc in $\mathbb{R}^2$ centered at ~~that pt~~ a pt in $(0,1)$ leaves $(0,1)$.

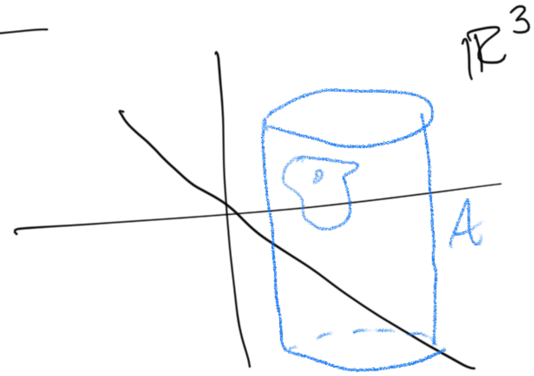
<u>Def</u>: $A \subset \mathbb{R}^n$, $x \in \mathbb{R}^n$, a <u>neighborhood of x relative to A</u> is a set of form $A \cap D^n(x, r)$ for some r.

<u>Ex</u> $n=2$, $\mathbb{R}^2$
$A = \mathbb{R}^1$
$x = (\frac{1}{2}, 0)$
rel nbhd: $A \cap B((1/2,0), r)$
$\mathbb{R}^1 \cap B((1/2,0), r)$
$= (\frac{1}{2}-r, \frac{1}{2}+r)$ $(= D^1(\frac{1}{2}, r))$

<u>Def</u>: $A \subset \mathbb{R}^n$, $B \subset A$, say $x \in B$ is <u>interior relative to A</u> if there exists a nbhd of x relative to A that is contained in B.
Say B is <u>relatively open in A</u> if every pt of B is interior relative to A.

<u>Ex</u>: $A = \mathbb{R}^1$, $n=2$
$B = (0,1) \subset \mathbb{R}^1 \subset \mathbb{R}^2$
B is relatively open in $A = \mathbb{R}^1$

Ex

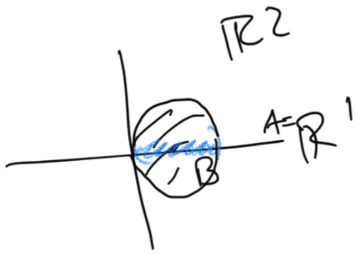


Thm: If $B \subset A \subset \mathbb{R}^n$, then B is open relative to A iff

$$B = A \cap U,$$

for U some open subset of $\mathbb{R}^n$.

Ex.



$$U = B\left((1/2, 0), 1/2\right)$$