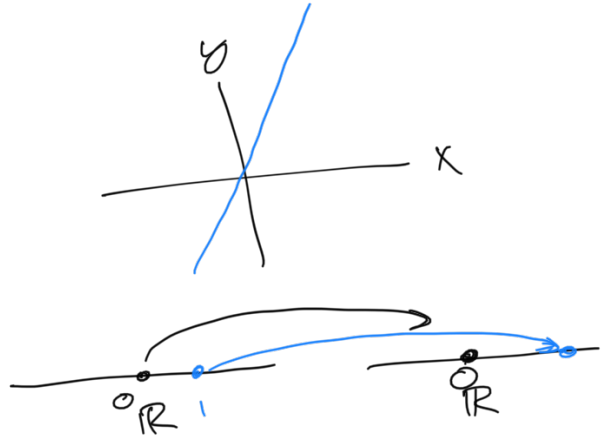


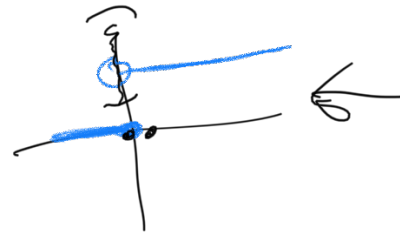
Continuity

Examples of Functions

① $F(x) = 2x$
 $F: \mathbb{R} \rightarrow \mathbb{R}$
 Domain Range



② $g: \mathbb{R} \rightarrow \mathbb{R}$
 $g(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$



$F: D \rightarrow R$ (for now $D \subset \mathbb{R}^n$, $R \subset \mathbb{R}^m$)
 Domain Range
 (target)

$A \subset D$
Image $F(A) := \{f(x) : x \in A\}$ "image of A under f"
 $F(D)$ "image of f"

Ex: image of g from above
 $g(\mathbb{R}) = \{0, 1\}$

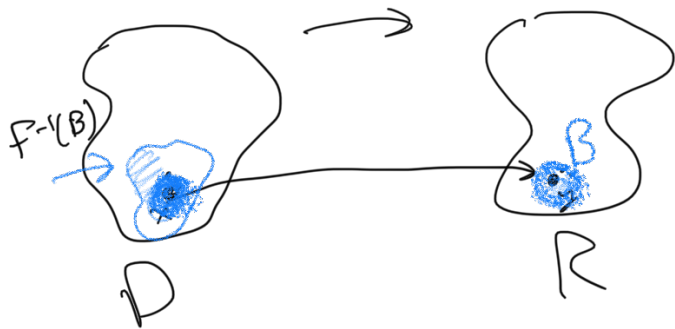
Pre-image $B \subset R$

$F^{-1}(B) := \{x \in D : f(x) \in B\}$

EX: $g^{-1}(\{1/2, 2\}) = \{x : x > 0\}$
 $= (0, \infty)$

Def: $F: D \rightarrow R$, say F is continuous if for $B \subset R$ the pre-image

any open set B in R ,
 $F^{-1}(B)$ is open in D .
relative to



Ex: (1) F from above is continuous

$$f(x) = 2x$$

Enough to show: $F^{-1}([a, b])$ is open

$$F^{-1}([a, b]) = \left(\frac{a}{2}, \frac{b}{2}\right)$$

which is open

Non-ex (2) $g(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$ not continuous

$$g: \mathbb{R} \rightarrow \mathbb{R}$$

To show this, need to find some open

$B \subset \mathbb{R}^1$ s.t. $g^{-1}(B)$ not open

Take $B = \left(\frac{1}{2}, \frac{3}{2}\right)$

$$g^{-1}(B) = \{x : x > 0\} = (0, \infty)$$

Take $B = \left(-\frac{1}{2}, \frac{1}{2}\right)$ (contains 0, ^{but} not 1)

$$g^{-1}(B) = \{x : x \leq 0\} = (-\infty, 0]$$

not open! ($0 \in g^{-1}(B)$ is not an interior pt)