Metric spaces and topologies

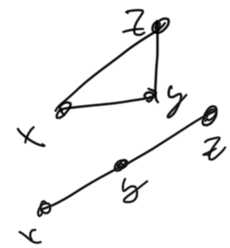
Metric Space: Set w/ a notion of distance

Def: A metric space is set X and function $d: X \times X \to \mathbb{R}_{\geq 0}$ s.t.

(i) $d(x,y) = 0$ iff $x = y$

(ii) $d(x,y) = d(y,x)$ Transitivity

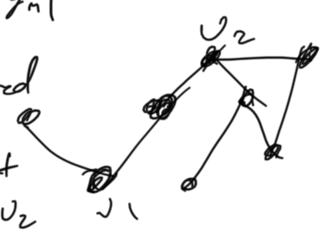
(iii) $d(x,z) \leq d(x,y) + d(y,z)$
Triangle Inequality



Ex: $X = \mathbb{R}^n$, $d(x,y) = \sqrt{(x_1-y_1)^2 + \dots + (x_n-y_n)^2}$

Ex: $X = \mathbb{R}^n$
$d(x,y) = |x_1 - y_1| + \dots + |x_n - y_n|$

Ex: X = any graph that is connected
$d(v_1, v_2)$ = # edges in shortest path between v_1, v_2



$d(v_1, v_2) = 2$

Notion of a topology

$\subset \mathcal{P}(X)$ $\mathcal{T}$ are "open sets"

☆ Defn: X set, $\mathcal{T}$ = collection of subsets of X
then $\mathcal{T}$ gives a topology if

(i) $X, \emptyset$ are in $\mathcal{T}$

(ii) Union of any collection of elts of $\mathcal{T}$ is in $\mathcal{T}$

(iii) intersection of any finite collection of elts of $\mathcal{T}$ is in $\mathcal{T}$

Ex: $X = \mathbb{R}^n$, $\mathcal{T} = \{\text{open sets in } \mathbb{R}^n\}$

Ex: X any set, $\mathcal{T} = \mathcal{P}(X)$ i.e. all subsets of X
"discrete topology"

" discrete top. "

Ex: $X =$ any set, $\mathcal{T} = \{X, \phi\}$

" indiscrete topology "

Ex: $X \subset \mathbb{R}^n$, $\mathcal{T} = \{S \subset X : S$ is relatively open in $X\}$