Subspace, continuity, connectedness

Def: (X, T) top. space, $A \subset X$, then the subspace topology T_A on A consists of all sets of form $U \cap A$, $U \in T$

Eg. $X = \mathbb{R}$, T = standard top.

$A = \{0, 1\}$

T_A = all subsets of A

i.e. discrete topology

$U \cap A = \{0\}$

$U' \cap A = \{1\}$

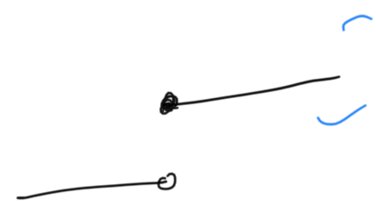
Def (X, T_X), (Y, T_Y) top. spaces, $f: X \to Y$

Say f is continuous (wrt T_X, T_Y) if for all $V \in T_Y$, $f^{-1}(V) \in T_X$.

Non Eg. (1) $X = \mathbb{R}$, T_X = standard top.

$Y = \mathbb{R}$, T_Y = standard top

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{if } x \geq 0 \end{cases}$$



f is not continuous: $f^{-1}\left(\left(1/2, 3/2\right)\right) = \{x : x \geq$

$[0, +\infty)$

open in $\mathbb{R}$, i.e. in T_Y

not open i

i.e. not i

T_X

Eg. (2) $X = \mathbb{R}$, T_X = all subsets of $\mathbb{R}$ (discrete top.)

$Y = \mathbb{R}$, T_Y = standard

$$f(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

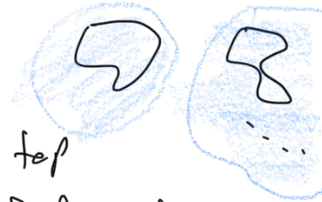
f is continuous: $f^{-1}(V)$ = same subse

$V \in T_Y$

of $X = \mathbb{R}$,

lies in T_X

Def Say (X, T_X) is **connected** if it **cannot** be written as the union of disjoint non-empty open subsets.



Eg ① $X = [0,1]$, T_X = standard top (i.e. subspace top inherited from $\mathbb{R}^1$)

Proved that (X, T_X) is connected

Non Eg. ① $X = [0,1]$, T_X discrete topology

(X, T_X) **not** connected.

$$X = [0, 1/2] \cup (1/2, 1]$$

both in T_X