Separation axioms

Def A top. space is T_0 if $\forall x, y \in X$ w/ $x \neq y$, $\exists$ open subset U containing exactly one of x, y.

Eg: ① $\mathbb{R}^n$

② $X = \{0, 1\}$
$\mathcal{T} = \{\emptyset, X, \{0\}\}$

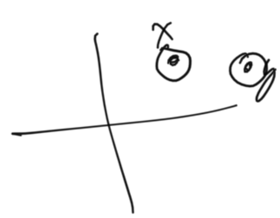
$x = 0$, $y = 1$

Non-ex

③ $X = \{0, 1\}$
$\mathcal{T} = \{\emptyset, X\}$ indiscrete topology
Not T_0

Def: A top. space is T_1 if $\forall x, y \in X$, $x \neq y$, $\exists$ open U_x, U_y s.t.
$x \in U_x$, $y \notin U_x$, $y \in U_y$, $x \notin U_y$

E.g. ④ $\mathbb{R}^n$



(5) Zariski top on $\mathbb{R}^n$

<u>Non-eg</u>

(6) Use (2) from above

Not T_1

No way choose U_y

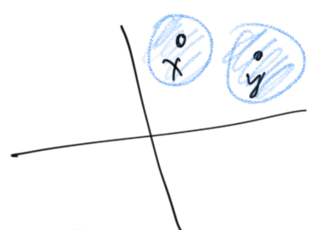
"Housed off"

☆ <u>Def</u> A top space is T_2 aka <u>Hausdorff</u> if $\forall x, y \in X$, $x \neq y$, there exists open U_x, U_y s.t. $x \in U_x$, $y \in U_y$, $U_x \cap U_y = \emptyset$

U_x: House for x

U_y: House y

<u>Eg.</u> (7) $\mathbb{R}^n$



<u>Non-e.g.</u> (8) Take (5) above

Also $T_{2\frac{1}{2}}$, T_3, $T_{3\frac{1}{2}}$, T_4

T_3: regular

T_4: normal