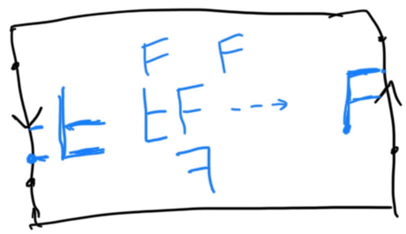
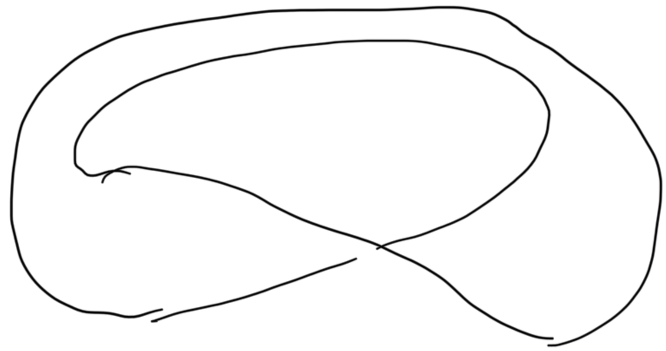
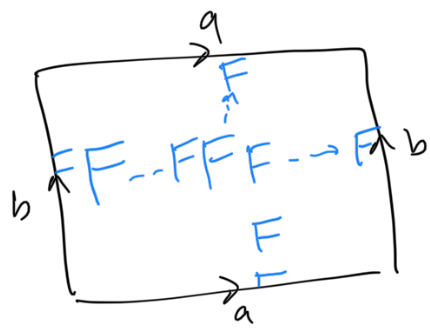
Surfaces - orientability and connect sum

Möbius strip

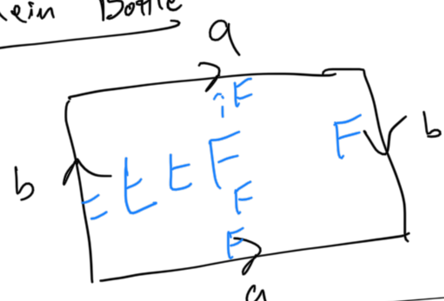




Torus



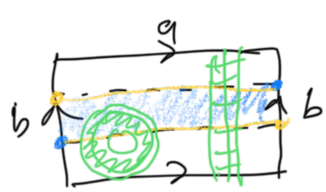
Klein Bottle



Defn: A surface S is non-orientable if it has a subspace homeomorphic to a Möbius strip.
Otherwise, S is orientable
(Topological Invariant)

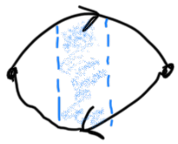
E.g. (1) Möbius strip is non-orientable

(2) Klein Bottle



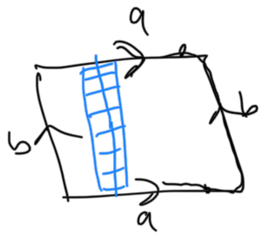
a

③ Projective Plane



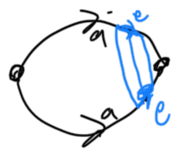
Non-example

④ Torus

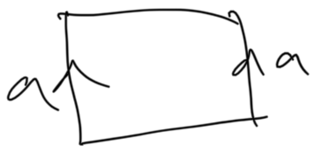


Orientable

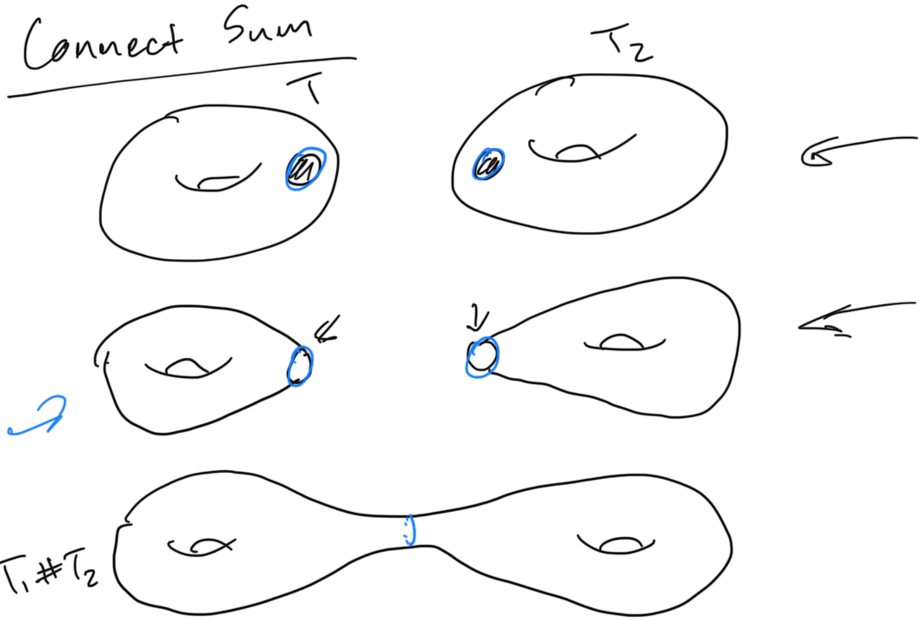
⑤ Sphere



⑥ Cylinder



Connect Sum



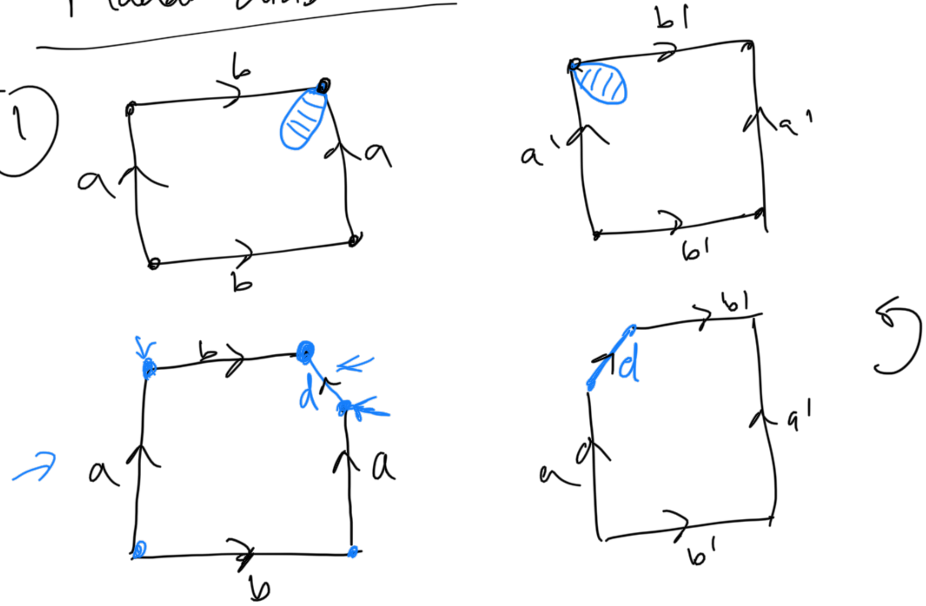
$T_1 \# T_2$

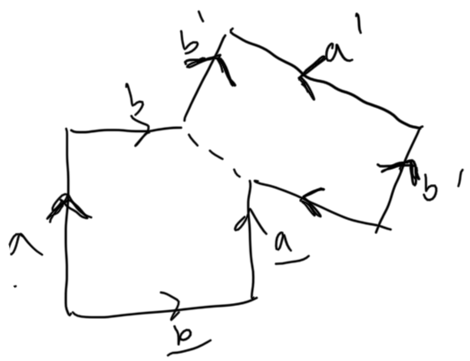
Connect sum

Connect sum: Start w/ S_1, S_2 surfaces remove a small (open) disk from each, and glue together the circular boundaries

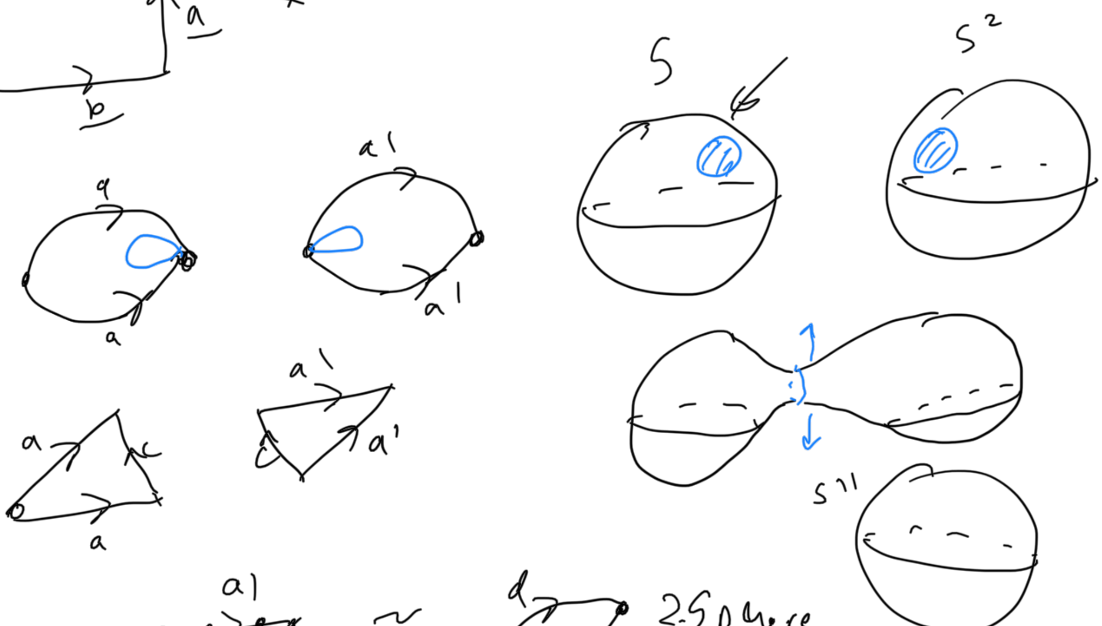
Planar diagrams

(1)

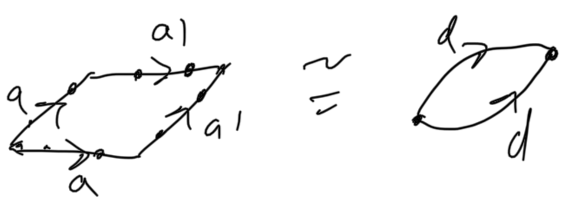




(2)



Glue



2-Sphere

Fact: $S^2 \# S = S$

2-sphere   any surface