

Thm (Classification of Surfaces) Any connected, compact surface (2-manifold without boundary) is homeomorphic to either

- (i) a sphere
- (ii) connect sum of n tori
- (iii) connect sum of n projective planes.

Prop 1: Every connected compact surface is a connect sum of tori and projective planes (includes 0 summands \rightarrow sphere).

PF of Prop 1:

Step 1: (Hardest part, will not prove)
Any connected, compact surface is homeomorphic to a surf. represented by planar diagram.

Claim: Any ¹ planar diagram with n edges is homeo. to connect sum of tori and projective planes.

Will prove Claim by induction on n .

Base Case: $n=1$

Two possibilities:



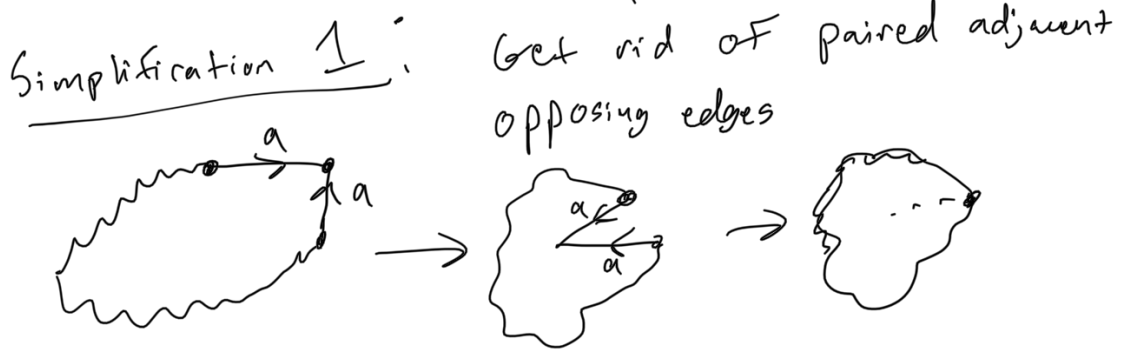
projective plane



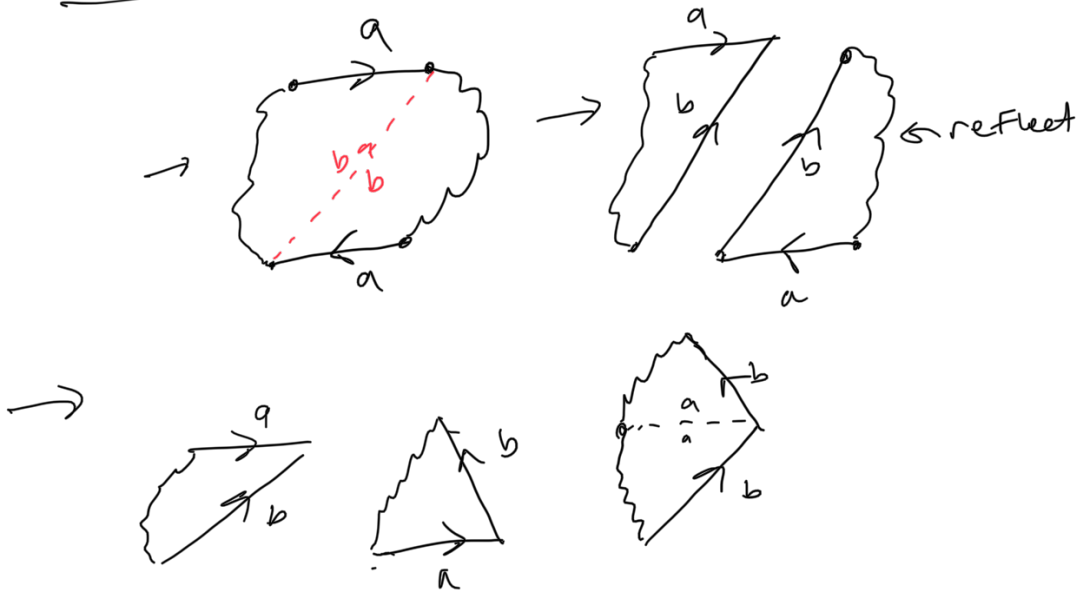
sphere



Inductive step: Assume for $n-1$ edges, want to prove for n edges.



Simp. 2 Bringing together twisted pairs



After Simp 1, 2 applied repeatedly, we get a planar diagram s.t.:

- no adjacent opposing pairs
- all twisted pairs are adjacent