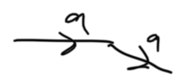
Proof of class. of surfaces, part 2

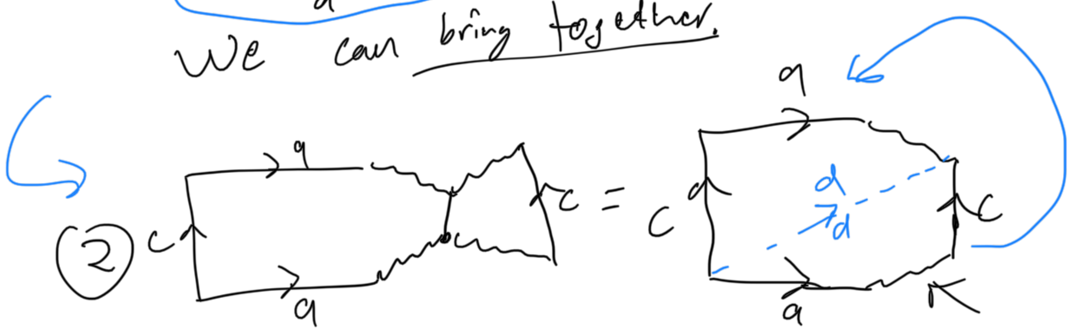
<u>Prop 1</u>: Every connected compact surface is a connect sum of tori and projective planes (includes 0 summands ⇝ sphere).

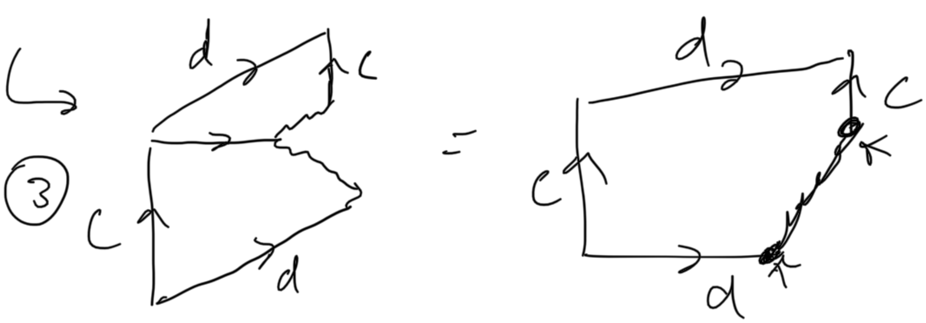
Arguing by induction on #edges. After applying Simp. 1, 2 from last time, get a planar diagram s.t.

- no <u>adjacent</u> opposing pairs 
- all <u>twisted pairs adjacent</u> 

<u>Case 1</u>: There exists an <u>interwoven</u> pair of opposing pairs.

We can <u>bring together</u>.

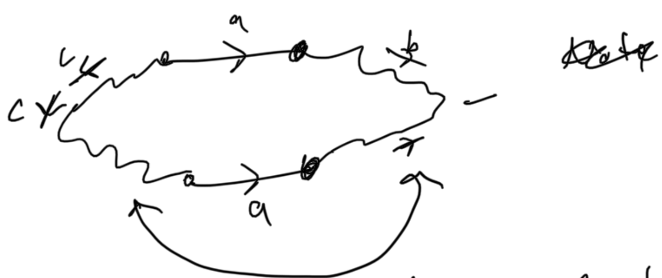




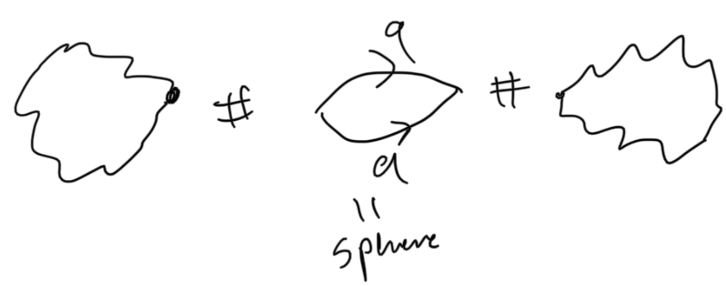
By induction is a connect sum of tori and proj. planes

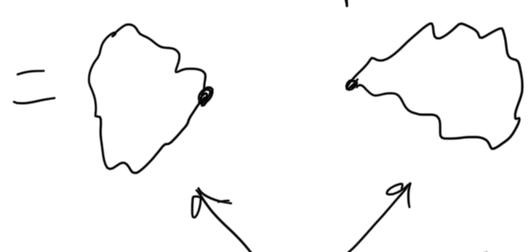
Hence so is original planar diagram

<u>Case 2</u>: There is an opposing pair, but no interwoven pairs of opposing pair.



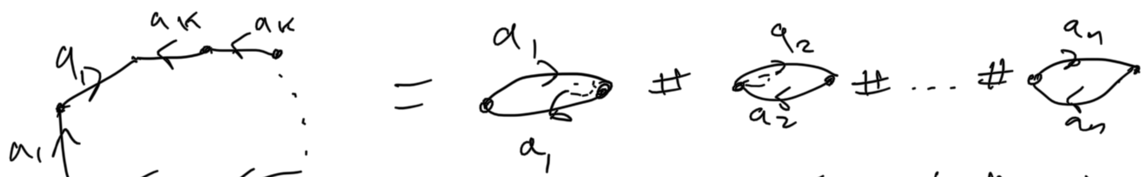
Note no pairs w/ one on left and one on right, s





Both have fewer edges than original, so each is connect sum of tori and proj. planes, by inductive hypothesis.

Case 3: No opposing pairs
(i.e. all pairs twisted)



Connect sum of projective planes

This completes the induction (proves Claim from last time), also completing pf of Prop 1. ∎