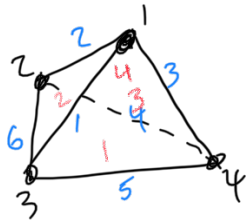
Platonic solids and Euler characteristic

Polyhedra for which all faces are congruent regular polygons, and the same # faces come together at each vertex.

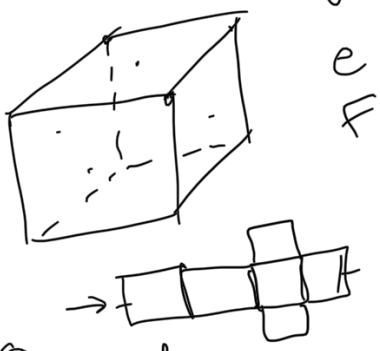
(1) Tetrahedron



Vertices = 4
edges = 6
Faces = 4

V - e + F
= 4 - 6 + 4 = 2

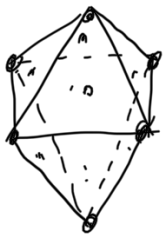
(2) Cube



v = 8
e = 12
F = 6

V - e + F
= 8 - 12 + 6 = 2

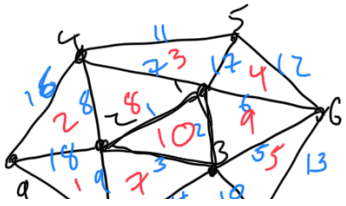
(3) Octahedron



v = 6
e = 12
F = 8

V - e + F
= 6 - 12 + 8 = 2

(4) Icosahedron

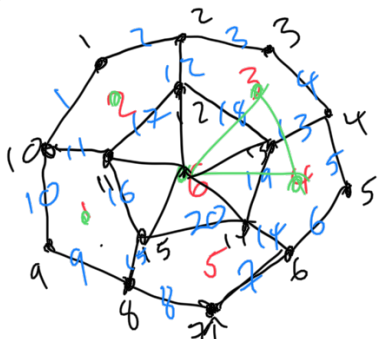


v = 12
e = 6 + 12 + 12 = 30
F = 10 + 10 = 20

V - e + F
= 12 - 30 + 20
= 2

looks identical on other side, but rotated

(5) Dodecahedron



looks identical on other side, but rotated

$v = 10+5+5 = 20$

$e = 10+10+10 = 30$

$F = 12$

$v-e+F =$

$20-30+12$

$= 2$

Thm: For any polyhedron homeo to sphere,

$v-e+F = 2$ ← Euler characteristic of sphere