Euler characteristic of CW-complexes

Reminder: CW-complex is a space built out cells of various dimension

Def The Euler characteristic $\chi(K)$ of a (Finite) CW-complex K is $e_0 - e_1 + e_2 - e_3 + \dots$

where $e_i = \#$ of i-cells

E.g. ① Graphs (1-dim'l CW-complexes)

$e_0 =$ vertices

$e_1 =$ edges

$\chi = e_0 - e_1 =$ vertices $-$ edges

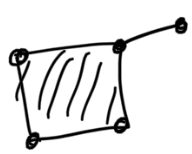
② Platonic solids (certain 2-dim'l CW-complexes)

$\chi = e_0 - e_1 + e_2$

$= v - e + F$

$\chi = \underline{4 - 6 + 4} = 2$

③



$e_0 - e_1 + e_2 = 5 - 5 + 1 = 1$

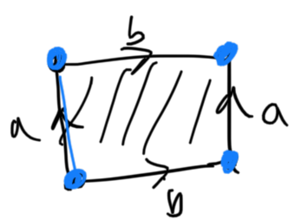
④

solid tetrahedron

$\chi = e_0 - e_1 + e_2 - e_3$

$= 4 - 6 + 4 - 1 = \underline{\underline{1}}$

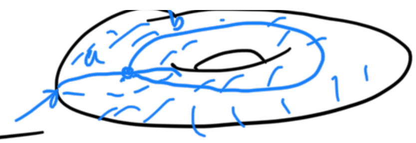
⑤



$e = 2,\ v = 1,\ F = 1$

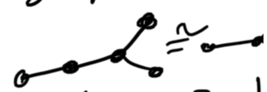
$\chi = v - e + F$

$= 1 - 2 + 1 = 0$



Last time: $G_1 \cong G_2$ homeomorphic graphs

then $\chi(G_1) = \chi(G_2)$.



$\rightarrow$ $\chi = 4-3=1$

CW-complex

Theorem: If $K_1 \cong K_2$ homeomorphic

then $\chi(K_1) = \chi(K_2)$

i.e. χ is a topological invariant.

Note: Can have $K_1 \not\cong K_2$, but $\chi(K_1) = \chi(K_2)$

e.g.  $\not\cong$ 

$\chi = 1$ $\qquad$ $\chi = 1$