Euler characteristic for planar diagrams

Thm If K_1, K_2 homeomorphic CW-complexes, then
$$\chi(K_1) = \chi(K_2).$$

Thm If P_1, P_2 planar diagrams representing homeomorphic surfaces, then $\chi(P_1) = \chi(P_2)$.

Proof: Study how χ changes under the operations used in proof of classification surfaces. Suffices to show χ doesn't change

$P_1 \rightsquigarrow Q_1$ connect sum of tori or projective plan

$P_2 \rightsquigarrow Q_2$ ''

$Q_1 = Q_2$ since P_1, P_2 represent homeo surfaces.

$$\chi(P_1) = \chi(Q_1) = \chi(Q_2) = \chi(P_2).$$

Steps in class. of surfaces

— Remove adjacent opposing edges

χ

χ'

$\chi = \chi'$

$f \rightsquigarrow f$

$e \rightsquigarrow e-1$

$v \rightsquigarrow v-1$

$\chi = v-e+f \rightsquigarrow (v-1)-(e-1)+f = v-e+f$

together twisted pairs

– Bringing

Get rid of a edge $\quad e \rightsquigarrow e-1+1 = e$

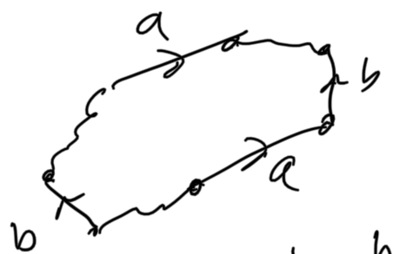
Add b edge

$v \rightsquigarrow v$

$f \rightsquigarrow f$

$v - e + f \rightsquigarrow v - e + f$ ✓

– Bringing together interwoven pairs of opposing pairs



Need two moves to bring together, neither changes v, e, f

– Opposing edge, no interwoven

no identifications between an edge on left and one on right

$v \rightsquigarrow v-1$

$e \rightsquigarrow e-1$

$f \rightsquigarrow f$

$v - e + f \to (v-1) - (e-1) + f = v - e + f$

$\blacksquare$