Euler characteristic and surfaces

Note: (1) $\chi(nT) = 2-2n$, $\quad$ T terms

$$nT = \underbrace{T \# \cdots \# T}_{n}$$

(2) $\chi(m\mathbb{P}^2) = 2-m$

Since χ is a topological invariant,

$n_1 T \not\cong n_2 T$ $\quad$ if $n_1 \neq n_2$

since $2-2n_1 \neq 2-2n_2$

and $m_1 \mathbb{P}^2 \not\cong m_2 \mathbb{P}^2$ if $m_1 \neq m_2$

But $\chi(T) = 0 = \chi(\mathbb{P}^2 \# \mathbb{P}^2)$ $\quad$ orientable $\quad$ non-orientable

(recall $\mathbb{P}^2 \# \mathbb{P}^2$ is Klein bottle)

<u>Thm</u>: If S_1, S_2 are compact, connected surfaces (without boundary), then $S_1 \cong S_2$ iff $\chi(S_1) = \chi(S_2)$ <u>and</u> both are orientable or both non-orientable.

<u>Recall</u>: A planar diagram gives a non-orientable surface iff it has at least one twisted pair of edges:

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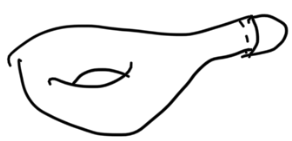
Surfaces with boundary

Thm If S_1, S_2 are compact, connect surfaces with boundary, then $S_1 \cong S_2$ iff $\chi(S_1) = \chi(S_2)$, both are orientable or both non-orientable, and S_1, S_2 have same number of boundary components.



WARNING: