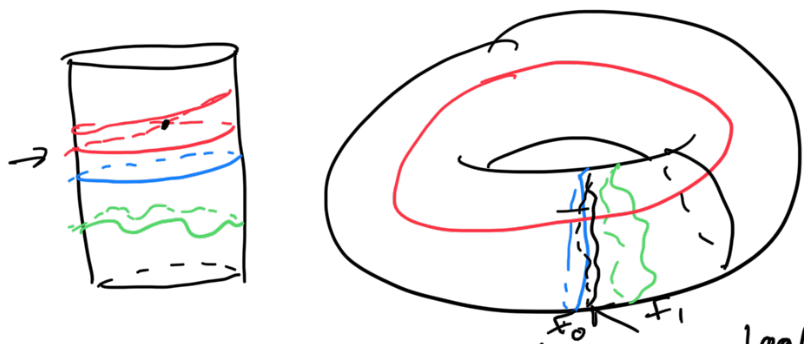
Intro to homotopy



In both, blue and green loops homotopic, while blue and red are not.

We think of a loop on Y as a map

$\gamma : S^1 \to Y$

$\uparrow$ circle

Def Let X, Y topological spaces, and $f, g : X \to Y$ continuous functions. Then f is homotopic to g (write $f \sim g$) if there exists a family $f_t : X \to Y$, $t \in [0,1]$, s.t.

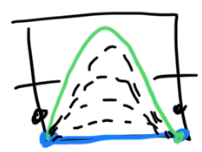
(i) $f_0 = f, \; f_1 = g$

(ii) $F : X \times [0,1] \to Y$, given by $F(x,t) := f_t($ is continuous

E.g. $X = S^1$, $Y =$ cylinder $[0,1] \times [0,1] / \sim$

$\quad \; = [0,1]/\sim \quad 0 \sim 1$

$(0,y) \sim (1,y)$

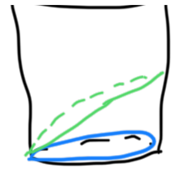


$f : X \to Y$

$f(x) = (x, 0) \quad \forall x$

$g : X \to Y$

$g(x) = \left(x, \frac{1}{4} - \underline{\left(x - \frac{1}{2}\right)}^2\right)$



Claim: f, g homotopic

$F(x,0) = (x, \underline{0})$

$F(x,1) = \left(x, \underline{\underline{\frac{1}{4} - \left(x - \frac{1}{2}\right)^2}}\right)$

Take $F(x,t)$

$$= \left(x, t\left(\frac{1}{4} - \left(x - \frac{1}{2}\right)^2\right)\right)$$

Cts, satisfies (i). So f, g homotopic.