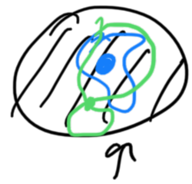
Homotopy equivalence

Note: Notion of $f, g : X \to Y$ being homotopic is an equivalence relation

Def: A space X is path-connected if $\forall x_1, x_2 \in X$, $\exists \gamma : [0,1] \to X$ continuous s.t. $\gamma(0) = x_1$, $\gamma(1) = x_2$



Def A space X is simply connected if it is (i) path-connected, and (ii) every continuous map $f : S^1 \to X$ is homotopic to a constant map (null-homotopic) $[g : S^1 \to X$, $g(s) = x_0$ $\forall s \in$

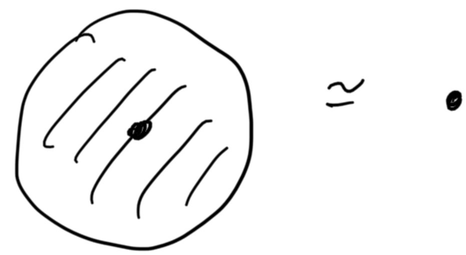
E.g. ① Disc D^n is simply connected

② S^2 two-sphere

Non-e.g ③ Annulus/cylinder not simply connected

Def: X, Y spaces are homotopy equivalent if $\exists\, f: X \to Y$ continuous, and continuous map $g: Y \to X$ s.t. $f \circ g \simeq id_Y$ ← identity $Y \to Y$
homotopic
and $g \circ f \simeq id_X$.

Eg: (1) $D^n \simeq pt.$
homotopy equivalence



$f: D^n \to pt$, constant $f(x) = pt$ $\forall x \in D^n$

$g: pt \to D^n$ $g(pt) = 0$

$f \circ g = id_{pt}$

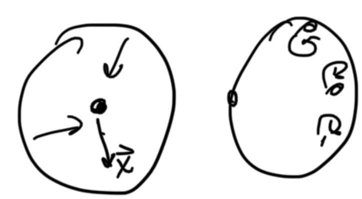
$g \circ f: D^n \to D^n$, $g \circ f(x) = 0$ $\forall x \in D^n$

Note $g \circ f \neq id_{D^n}$

But $g \circ f \simeq id_{D^n}$

$h_t: D^n \to D^n$, $t \in [0,1]$

Want: $h_0 = g \circ f$
$h_1 = id_{D^n}$



$h_t(x) = tx$
$h_0(x) = 0 \cdot x = 0$ $\forall x \in D^n$
$h_1(x) = 1 \cdot x = x$ $\forall x \in D^n$