Brouwer fixed point theorem

Last time: simply connected means any loop is contractible

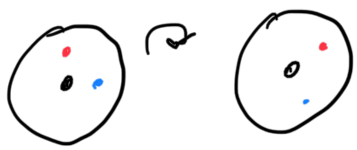
E.g. 

circle

Thm S^1 is not simply connected

B^2

Thm (Brouwer) Let B^n closed n-dimensional ba[ll]
$f: B^n \to B^n$ continuous. Then f has a fixed point, i.e. $\exists x \in B^n$ s.t. $f(x) = x$.

E.g. ① rotation 

② 

$f: LI \to LI$

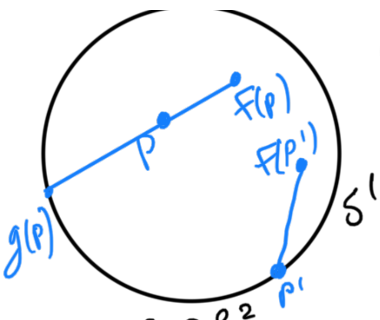
$p \mapsto$ pt on map that represents p

Brouwer $\Rightarrow f$ has fixed pt x

In HW, gave pf of $n=1$ case using Intermediate Value Thm.

Pf ($n=2$ case)
Suppose otherwise, ie f does not have a f.p.

$: B^2 \to S^1$



$f: B^2 \to B^2$

Get new map g:

$p \mapsto$ intersection of ray $\overrightarrow{f(p)\,p}$ w/ boundary circle

g is continuous (relies on f cts)

If $x \in S^1$, $g(x) = x$.

(g is a "retraction" from B^2 to S^1

Let $\gamma : S^1 \to S^1$ continuous that is not contractible (since S^1 is not simply connected)

But γ is contractible as a loop in B^2 (since B^2 is simply connected)

$p \in S^1$

So $\exists\, \gamma_t : S^1 \to \underline{\underline{B^2}}$, $t \in [0,1]$ continuous

s.t. $\gamma_0 = \gamma$, $\gamma_1 =$ constant map $x \mapsto p$

Now consider

$g \circ \gamma_t : S^1 \to \underset{\sim}{S^1}$ continuous.

$S^1 \xrightarrow{\gamma_t} B^2 \xrightarrow{g} S_1$ $\qquad g \circ \gamma_0 = \gamma_0 = \gamma$

$g \circ \gamma_1 = \gamma_1 =$ constant map

So $g \circ \gamma_t$ gives homotopy in S^1 been γ and constant map.

But γ is not contractible in S^1 $\times$

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