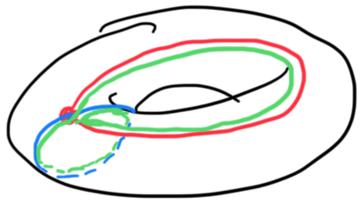
Based loops and composition

Want: invariants for topological spaces (e.g. Euler characteristic)

Loops: (up to homotopy)

Can compose loops based at same pt

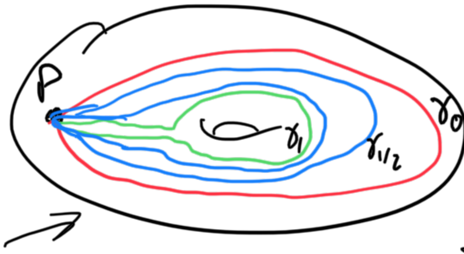


Based loops on X, base point $p \in X$

$\gamma : [0,1] \to X$

$\gamma(0) = p = \gamma(1)$

Def: γ, γ' based loops on (X,p) are homotopic if $\exists\, \gamma_t$ based loops on (X,p), for $t \in [0,1]$ s.t. $\gamma_0 = \gamma$, $\gamma_1 = \gamma'$ and γ_t depends continuously on t.



Def: If γ based loop on $(X,p$

$[\gamma] := \{\gamma' : \gamma'$ is homotopic to γ

"homotopy class of γ"

Def: Let X be space, $p \in X$,

$\pi_1(X,p) = \{[\gamma] : \gamma$ based loop denote $\pi_1(X,p) = \{[\ldots] \ldots$ on $(X,p)\}$

"Fundamental group" (as a set).

E.g. ①

Any based loop γ on (B^2, p) is homotopic to constant loop γ_p at p

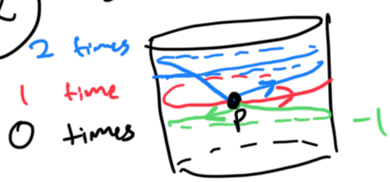
$[0,1] \ni x \longmapsto p$

So just one homotopy class $[\gamma_p]$

$\pi_1(X,p) = \{[\gamma_p]\}$

"trivial" one element

② Cylinder C



$\pi_1(C,p) = \{\ldots, -2, -1, 0, 1, 2, 3, \ldots$

$= \mathbb{Z}$

$p \in X$, $q \in Y$

<u>Note</u>: If X, Y are homeomorphic, then $\pi_1(X,p)$ has same cardinality as $\pi_1(Y,q)$

(so B^2 <u>not</u> homeomorphic C)