

Homework 01: MATH 4180

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

Note: This may not be representative of a typical homework, since we haven't covered much material yet.

1. Show that if z is a root of a polynomial with *real* coefficients, then \bar{z} is also a root (i.e. the roots come in conjugate pairs).
2. Does every polynomial with real coefficients and odd degree have a real root?
3. Recall that given $z \in \mathbb{C}$, we define a map $\phi_z : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, by $w \mapsto zw$, where we are using the standard identification of \mathbb{R}^2 with \mathbb{C} . Does every linear map $\phi : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ equal ϕ_z for some $z \in \mathbb{C}$?
4. Express all complex solutions of the equation $z^6 = 1$ in terms of trig functions. Draw a picture of the solutions.
5. Prove that if θ is not a multiple of π , then

$$\sin \theta + \sin 2\theta + \cdots + \sin n\theta = \frac{\sin \left(\frac{n+1}{2}\theta\right) \sin \left(\frac{n}{2}\theta\right)}{\sin(\theta/2)}.$$

(Hint: geometric series)