

Homework 2: MATH 4180

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

1. Given $a, b \in \mathbb{C}$, consider the function $f_{a,b} : \mathbb{C} \rightarrow \mathbb{C}$ given by $z \mapsto a \cos z + b \sin z$. Do there exist a, b , not both 0, such that $f_{a,b}$ is a bounded function?
2. What are the images of horizontal and vertical lines under $z \mapsto \cos z$?
3. Find all possible values of 3^i .
4. Let $U \subset \mathbb{C}$ be open and consider a function $f : U \rightarrow \mathbb{C}$. Show that f is continuous (according to the limit definition discussed in class) iff $f^{-1}(V)$ is open for any open set $V \subset \mathbb{C}$.
5. Let $f(z), g(z)$ be polynomials (with complex coefficients), neither of which is the zero polynomial. Show that

$$\lim_{z \rightarrow \infty} f(z)/g(z)$$

always exists, where we interpret values as lying in the Riemann sphere $\widehat{\mathbb{C}}$ (which will mean that the value of the limit could be a finite complex number or ∞).

6. Does $\lim_{z \rightarrow 0} e^{1/z}$ exist in $\widehat{\mathbb{C}}$?