Homework 3: MATH 4180

Collaboration Policy : You may, in fact are encouraged to, work on the problems with other students. You must write up your solutions by yourself.

1. Let $f : \mathbb{C} \to \mathbb{C}$ be given by $f(z) = |z|$. Determine the set of points in $\mathbb{C}$ at which f is complex differentiable.
2. Consider the function $f : \mathbb{C} \to \mathbb{C}$ given by $f(z) = 4z^5 + 5z^4$. Determine the set of points in $\mathbb{C}$ at which f is conformal.
3. Let $f : \mathbb{C} \to \mathbb{C}$ be a holomorphic function. Show that $z \mapsto \overline{f(\bar{z})}$ is also a holomorphic function $\mathbb{C} \to \mathbb{C}$.
4. Given an example of a function $f : D(0,1) \to \mathbb{C}$ (recall that $D(0,1)$ is the open disc of radius 1 with center at 0) such that f is conformal everywhere on $D(0,1)$, but there exists a line segment in $D(0,1)$ whose image under f is not a line segment.
5. Find an example of a holomorphic function $f : D(0,1) \to \mathbb{C}$ that is conformal everywhere on $D(0,1)$ but f' is not.